

Nonlinear parametric optics

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Nonlinear parametric optics

1. Principles of three-wave mixing

- Nonlinear response
- Difference-frequency generation: conversion efficiency and acceptance
- Phase- and quasi-phase-matching

2. Parametric amplification, generation, and oscillation

- Parametric amplification and reconversion effects
- Parametric fluorescence
- Parametric oscillation, variations of OPO schemes

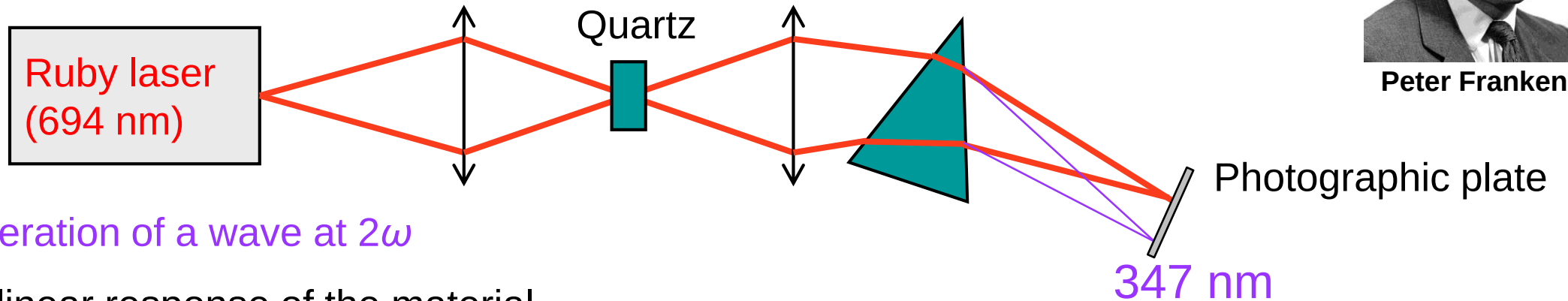
3. Implementations and related applications

- Continuous-wave regime
- Nanosecond regime, spectral content issues
- Picosecond to femtosecond regime

Conclusion

Birth certificate of nonlinear optics

Franken's experiment (1961)



Peter Franken

Generation of a wave at 2ω

Nonlinear response of the material

$$\cos(\omega t) \rightarrow \cos^2(\omega t) = \frac{1}{2} + \frac{1}{2} \cos(2\omega t)$$

Invention of laser $\rightarrow$ high intensities $\rightarrow$ high electric field in the material

3J, 1 ms, pulse
focused on 50 μm

$$\Pi = \frac{\epsilon_0 c}{2} E^2 \approx 10^{12} \text{ W/m}^2 \Rightarrow E \approx 10^7 \text{ V/m}$$

Franken's result... as published!

VOLUME 7, NUMBER 4

PHYSICAL REVIEW LETTERS

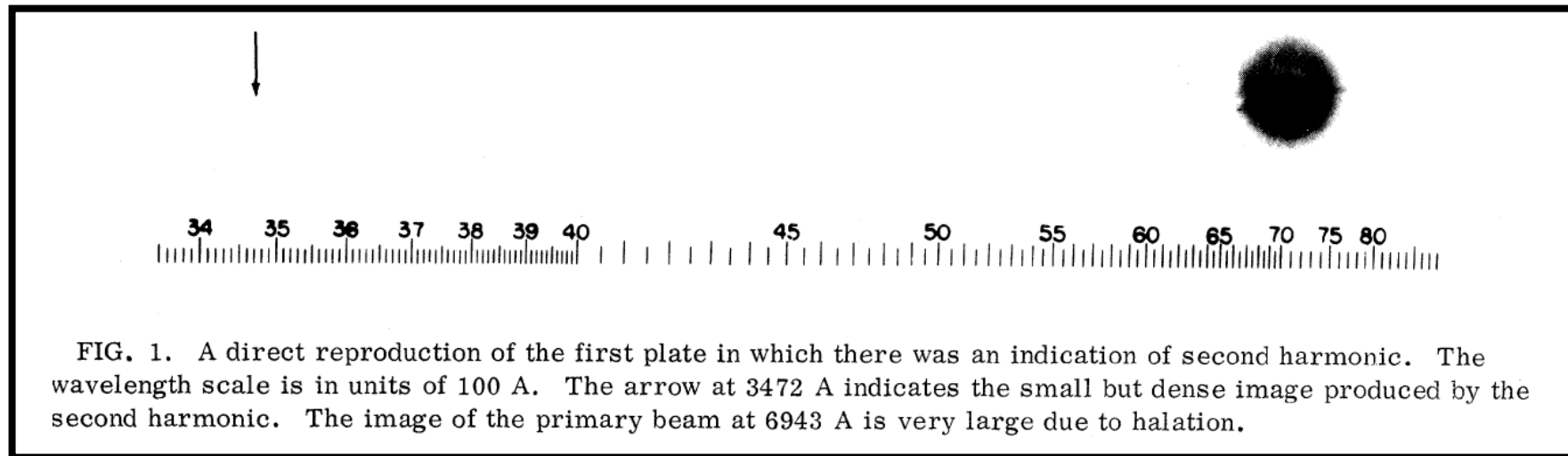
AUGUST 15, 1961

GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich

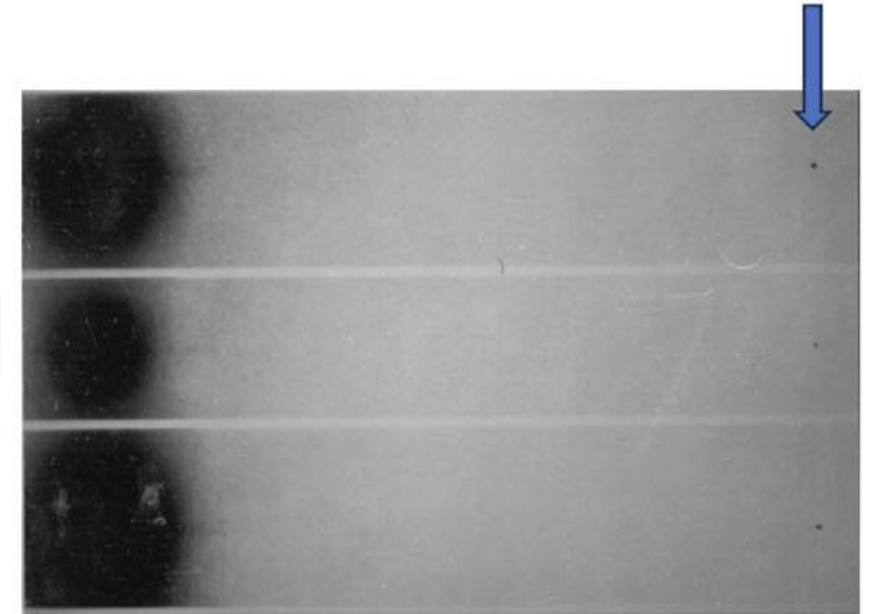
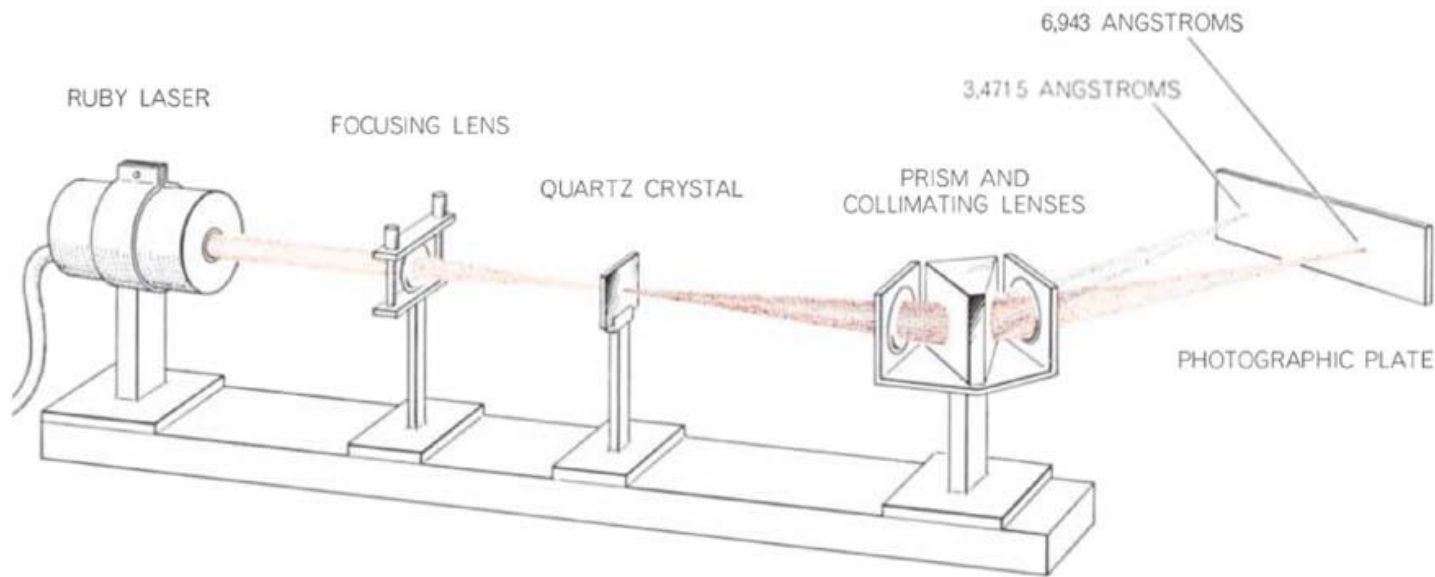
The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan

(Received July 21, 1961)



Invisible 2ω signal! The tiny spot was thought by the lithographers at Physical Review Letters to be a grain of dust, and was therefore eliminated from the published version of the article...

Franken's original photographic plates



Chimire & Reis, APL Photon. 11, 021102 (2026)

2ω signal is present!

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Nonlinear optical response of a medium to an intense light excitation

Apparition of high-order (nonlinear) terms in the expression of the dielectric polarization in a medium as a function of the electric field

$$\vec{P} = \varepsilon_0 \left(\underbrace{\chi^{(1)} : \vec{E}}_{\vec{P}_L} + \underbrace{\chi^{(2)} : \vec{E}\vec{E} + \chi^{(3)} : \vec{E}\vec{E}\vec{E} + \dots}_{\vec{P}_{NL}} \right)$$

Anisotropic medium → tensorial relation between $\vec{P}$ and $\vec{E}$

$\chi^{(1)}$: linear susceptibility (second-order tensor)

$\chi^{(n)}$: n^{th} order nonlinear susceptibility ($(n + 1)^{\text{th}}$ order tensor)

Relative orders of magnitude of the different orders of the susceptibility

Nonlinearity only appears if the laser field is not completely negligible compared to the intra-atomic field

$$E_{\text{at}} \approx 10^{11} \text{ V/m}$$

$$\Rightarrow P = \eta^{(1)} \frac{E}{E_{\text{at}}} + \eta^{(2)} \left(\frac{E}{E_{\text{at}}} \right)^2 + \eta^{(3)} \left(\frac{E}{E_{\text{at}}} \right)^3 + \dots$$

where coefficients $\eta^{(n)}$ are all of the same order of magnitude (valid off-resonance)

$$\Rightarrow \chi^{(n+1)} \ll \chi^{(n)}$$

In most cases (off resonance), development is limited to

- $\chi^{(2)}$ in non-centrosymmetric media  **quadratic nonlinear optics**
- $\chi^{(3)}$ in other media

Nonlinear mixing of two monochromatic waves

$$E_1(\vec{r}, t) = \frac{1}{2} E_1(\vec{r}) e^{-i\omega_1 t} + \text{c.c.} \quad E_2(\vec{r}, t) = \frac{1}{2} E_2(\vec{r}) e^{-i\omega_2 t} + \text{c.c.}$$

➔
$$P_3^{(2)}(\vec{r}, t) = \frac{1}{2} \varepsilon_0 \chi^{(2)} E_1(\vec{r}) E_2(\vec{r}) e^{-i(\omega_1 + \omega_2)t} + \text{c.c.} \quad \text{sum frequency}$$

$$+ \frac{1}{2} \varepsilon_0 \chi^{(2)} E_1(\vec{r}) E_2(\vec{r})^* e^{-i(\omega_1 - \omega_2)t} + \text{c.c.} \quad \text{difference frequency}$$
$$+ \frac{1}{4} \varepsilon_0 \chi^{(2)} E_1(\vec{r})^2 e^{-i2\omega_1 t} + \dots \quad \text{frequency doubling}$$

Coupled-mode equations

How waves 2 and 3 combine to produce wave 1:

$$\nabla^2 E_1(z,t) - \left(\frac{n_1}{c}\right)^2 \frac{\partial^2}{\partial t^2} E_1(z,t) = \frac{\mu_0 \varepsilon_0 \chi_2}{2} \frac{\partial^2}{\partial t^2} P_{NL,1}(z,t)$$

$$\text{avec } \omega_1 = \omega_3 - \omega_2$$

We introduce envelope functions:

$$\begin{aligned} \nabla^2 \left[\frac{1}{2} E_1(z) e^{-ik_1 z} + \text{c.c.} \right] + \left(\frac{n_1 \omega_1}{c} \right)^2 \left[\frac{1}{2} E_1(z) e^{-ik_1 z} + \text{c.c.} \right] = \\ - \frac{\mu_0 \varepsilon_0 \chi_2}{2} \omega_1^2 \left(E_2(z)^* e^{ik_2 z} E_3(z) e^{-ik_3 z} + \text{c.c.} \right) \end{aligned}$$

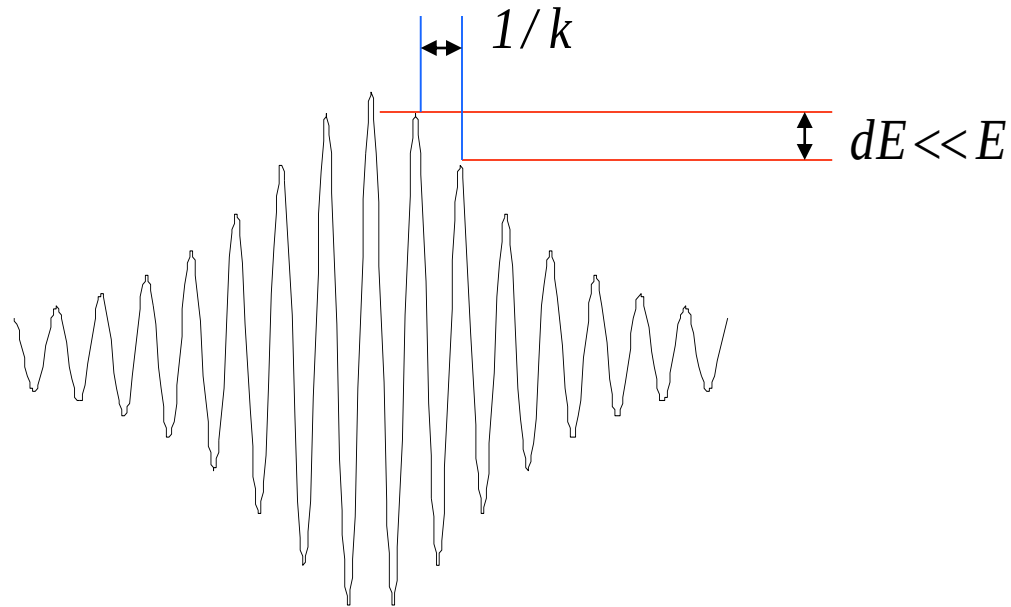
$$\cancel{\frac{d^2}{dz^2} E_1(z)} - 2ik_1 \frac{d}{dz} E_1(z) e^{-ik_1 z} - \cancel{k_1^2 E_1(z) e^{-ik_1 z}} + \cancel{\left(\frac{n_1 \omega_1}{c}\right)^2 E_1(z) e^{-ik_1 z}} =$$

$$- \mu_0 \varepsilon_0 \chi_2 \omega_1^2 \left(E_2(z) e^{ik_2 z} E_3(z) e^{-ik_3 z} \right)$$

Dispersion relation: $\omega_i = k_i \frac{c}{n_i}$

Envelope function approximation

$$\left| \frac{d^2 E_j}{dz^2} \right| \ll \left| k_j \frac{d E_j}{dz} \right|$$



$$-2ik_1 \frac{d}{dz} E_1(z) e^{-ik_1 z} = -\mu_0 \varepsilon_0 \chi_2 \omega_1^2 \left(E_2(z)^* e^{ik_2 z} E_3(z) e^{-ik_3 z} \right)$$

Coupled-mode equations



$\omega_3 - \omega_2 \rightarrow \omega_1$	$\frac{d}{dz} E_1 = -i \frac{\omega_1}{2n_1 c} \chi_2 E_3 E_2^* e^{-i\Delta k z}$
$\omega_3 - \omega_1 \rightarrow \omega_2$	$\frac{d}{dz} E_2 = -i \frac{\omega_2}{2n_2 c} \chi_2 E_3 E_1^* e^{-i\Delta k z}$
$\omega_1 + \omega_2 \rightarrow \omega_3$	$\frac{d}{dz} E_3 = -i \frac{\omega_3}{2n_3 c} \chi_2 E_1 E_2 e^{+i\Delta k z}$

Phase mismatch

$$\Delta k = k_3 - k_1 - k_2$$

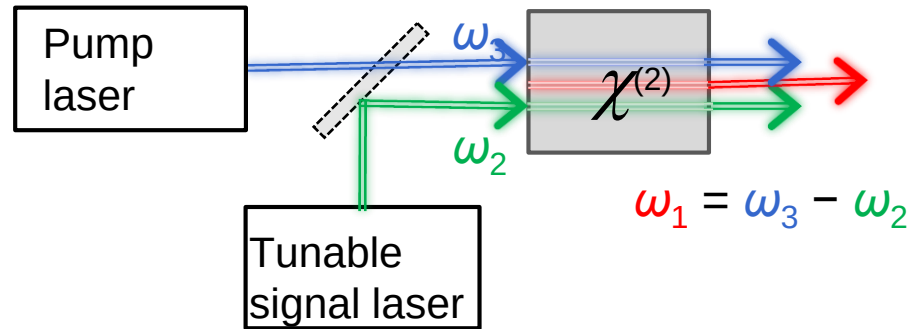
with $k = n(\omega) \omega / c$

Difference frequency generation (DFG)

- Two incident waves, ω_2 and $\omega_3 \rightarrow$ DFG at $\omega_1 = \omega_3 - \omega_2$
- We assume a low efficiency, $E_2(z) \approx E_2(0)$ et $E_3(z) \approx E_3(0)$
- Only one equation to solve

$$\frac{d}{dz} E_1 = -i \frac{\omega_1}{2n_1 c} \chi_2 E_{30} E_{20}^* e^{-i \Delta k z}$$

$$\Rightarrow |E_1(z)| = \frac{\omega_1}{2n_1 c} \chi_2 E_{30} E_{20}^* |z \operatorname{sinc}\left(\frac{\Delta k z}{2}\right)|$$



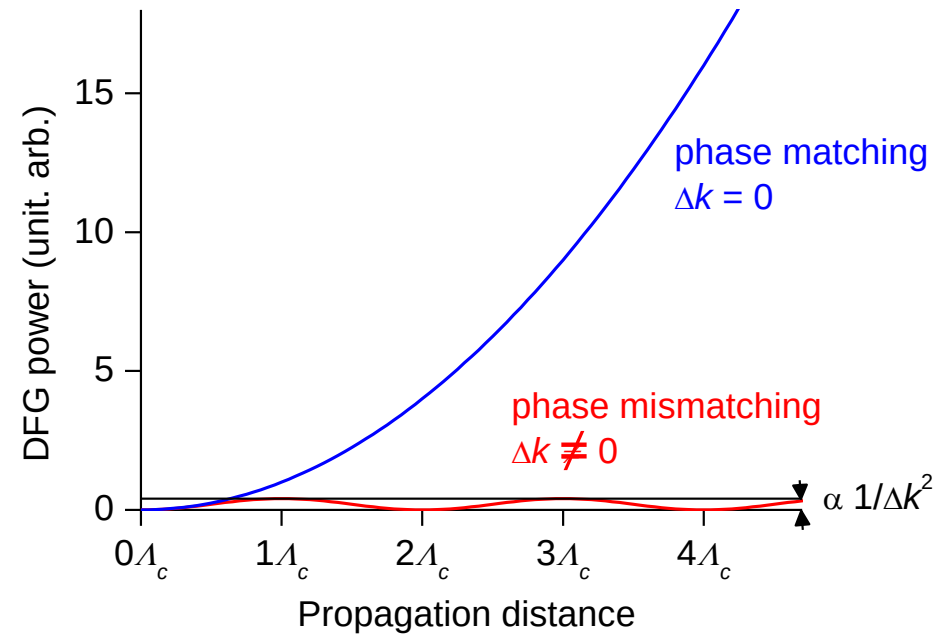
Conversion yield

$$I_1(z) = \frac{2}{\epsilon_0 c} \left(\frac{\omega_1 \chi_2 z}{2n_1 c} \right)^2 I_3 I_2 \operatorname{sinc}^2\left(\frac{\Delta k z}{2}\right)$$

Coherence length

$$I_1(z) \propto z^2 \operatorname{sinc}^2\left(\frac{\Delta k z}{2}\right) I_2 I_3$$

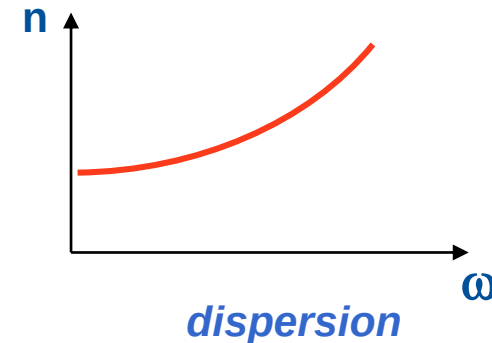
$$\Rightarrow I_1(z) \propto \frac{1}{\Delta k^2} \sin^2\left(\frac{\Delta k z}{2}\right) I_2 I_3$$



Maximum conversion for: $z = \Lambda_c$ with $\Delta k \Lambda_c = \pi$

Coherence length:

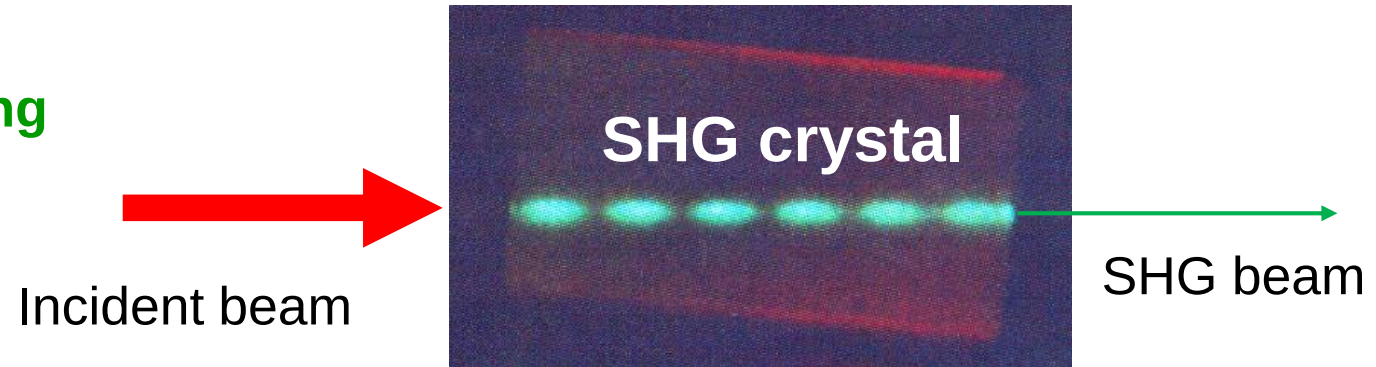
$$\Lambda_c = \frac{\pi}{\Delta k}$$



Practical illustration of the phase mismatch effect

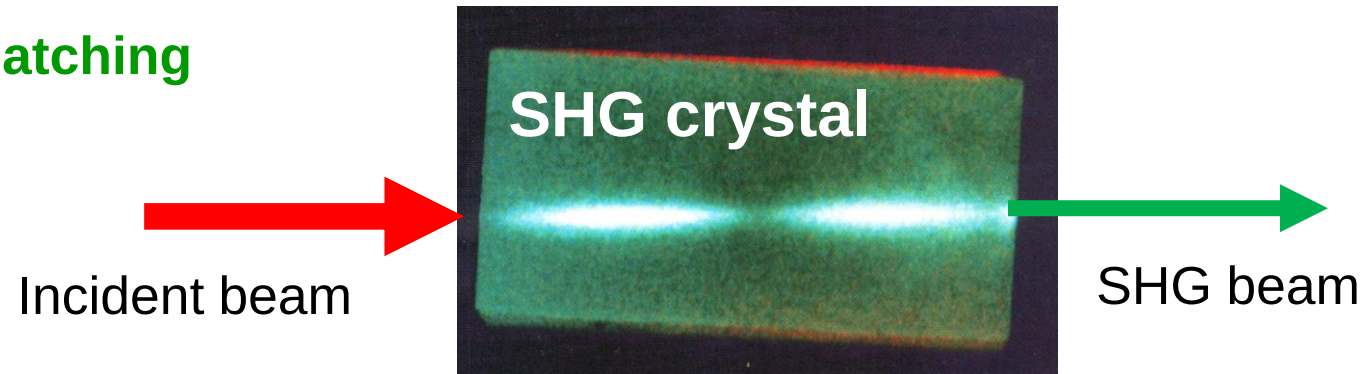
Case of SHG : $\omega + \omega \rightarrow 2\omega$

Far from phase matching



Source : Prof. R. Trebino (Georgia Tech)

Less far from phase matching



It can be seen that the SH beam is more intense the closer one gets to phase matching.



Being nearly phase-matched is sufficient: $\Delta k \leq \pi/L$

Phase-matching acceptance

Case of DFG (low conversion efficiency)

$$I_1(z) \propto z^2 \operatorname{sinc}^2\left(\frac{\Delta k z}{2}\right) I_2 I_3$$

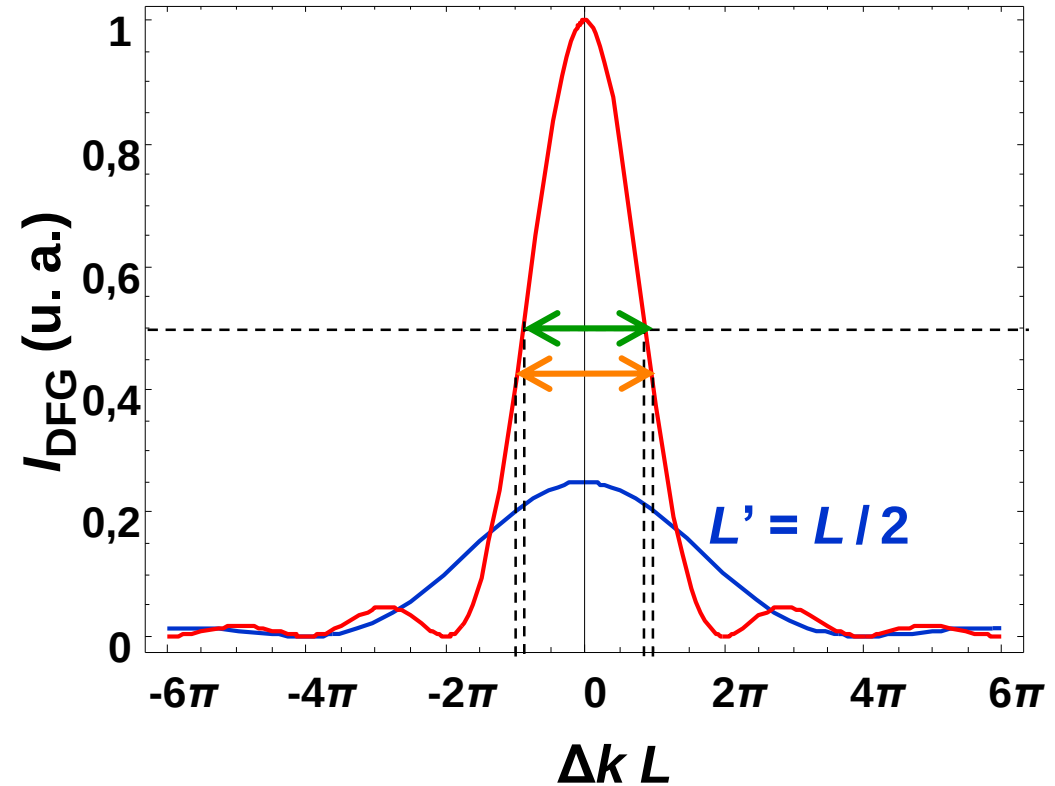
50% reduction in conversion for:

$$\Delta k_{BP} = 2,78/L$$

This is one of the definitions of acceptance bandwidth.

Another usual definition: $\Delta k_{BP} = \pi/L$

i.e. $L = L_c$



The tolerance in terms of phase matching is all the more critical as the length of the nonlinear medium increases.

Solution to one of the mysteries of 3-wave interaction

Of all triplets of wavelengths such as:

$$\omega_1 + \omega_1 = 2\omega_1 \quad \omega_2 + \omega_2 = 2\omega_2 \quad \omega_3 + \omega_2 \quad \omega_3 - \omega_2 \quad \omega_1 - \omega_2 \quad \omega_1 + \omega_2 = \omega_3 \quad \text{etc...}$$

which can exchange energy through parametric interaction,
only triplets such as

$$\omega_1 + \omega_2 = \omega_3$$

Conservation of energy

$$n(\omega_1)\omega_1 + n(\omega_2)\omega_2 \approx n(\omega_3)\omega_3 \Leftrightarrow \Delta k \approx 0 \quad \text{Phase matching}$$

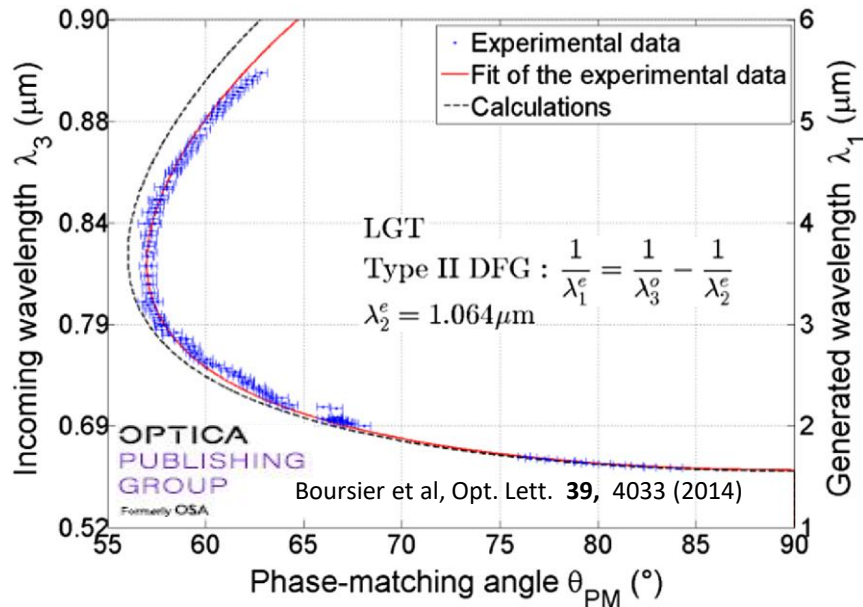
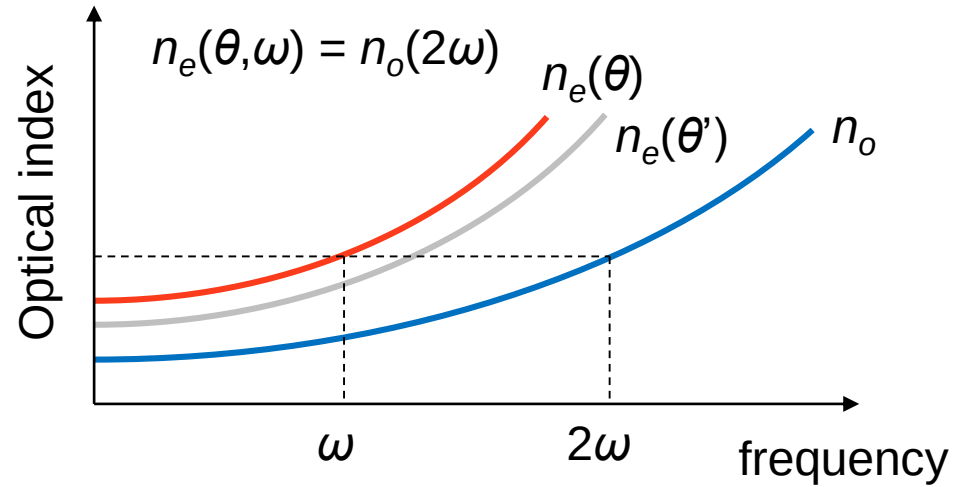
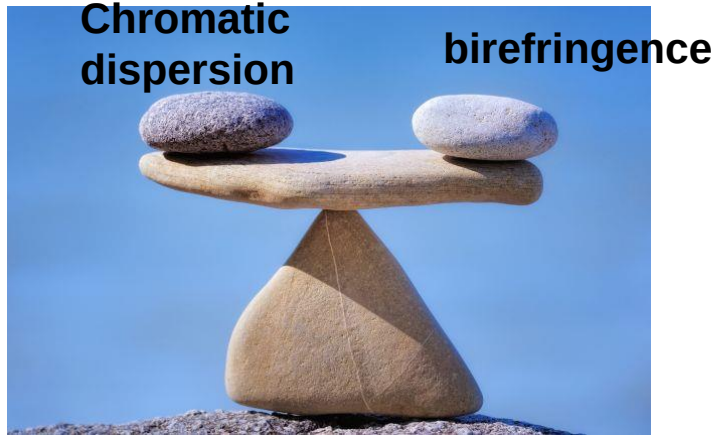
efficiently interact.

For a given ω_3 , a single (ω_1, ω_2) couple simultaneously satisfies these two equations (generally).



Importance of the phase-matching condition

Birefringence phase matching

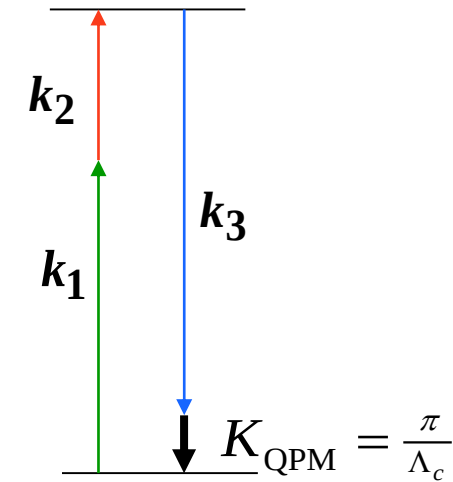
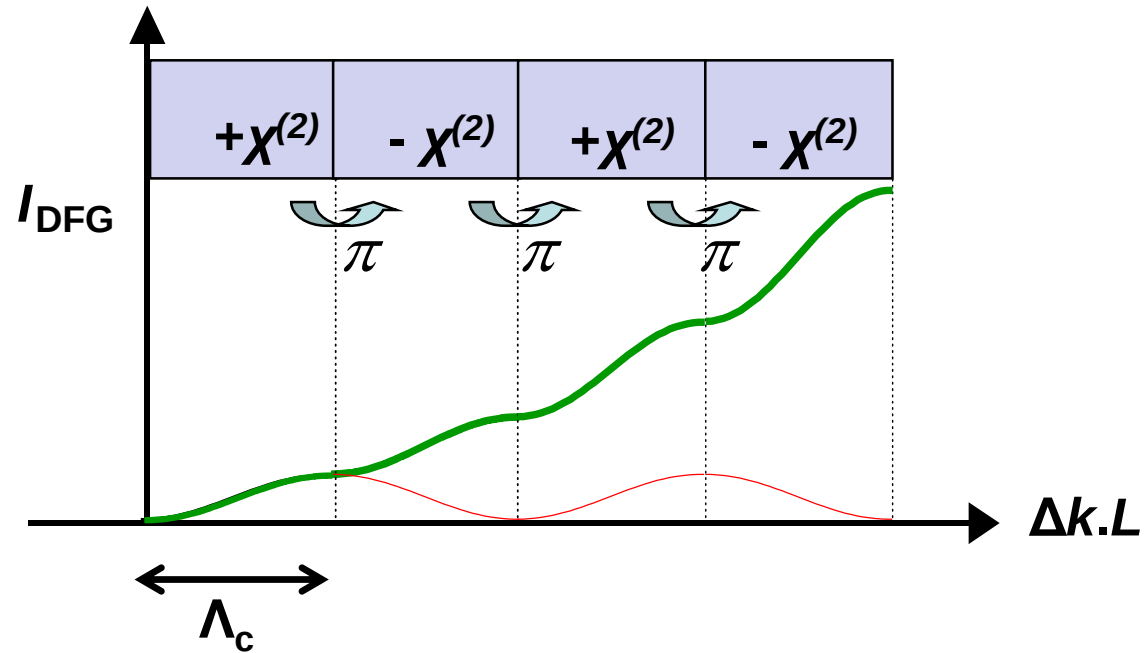


- Demonstrated since the 1960s
- Long the only effective method

Few limitations

- Only birefringent materials
- Double refraction
- Orientation generally different from that which maximises the nonlinear coefficient d_{eff}

Quasi phase matching



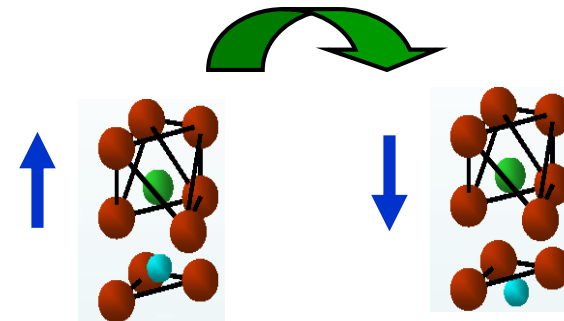
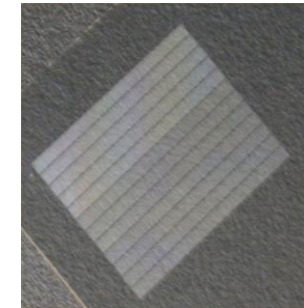
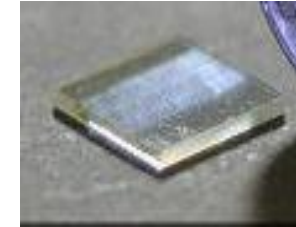
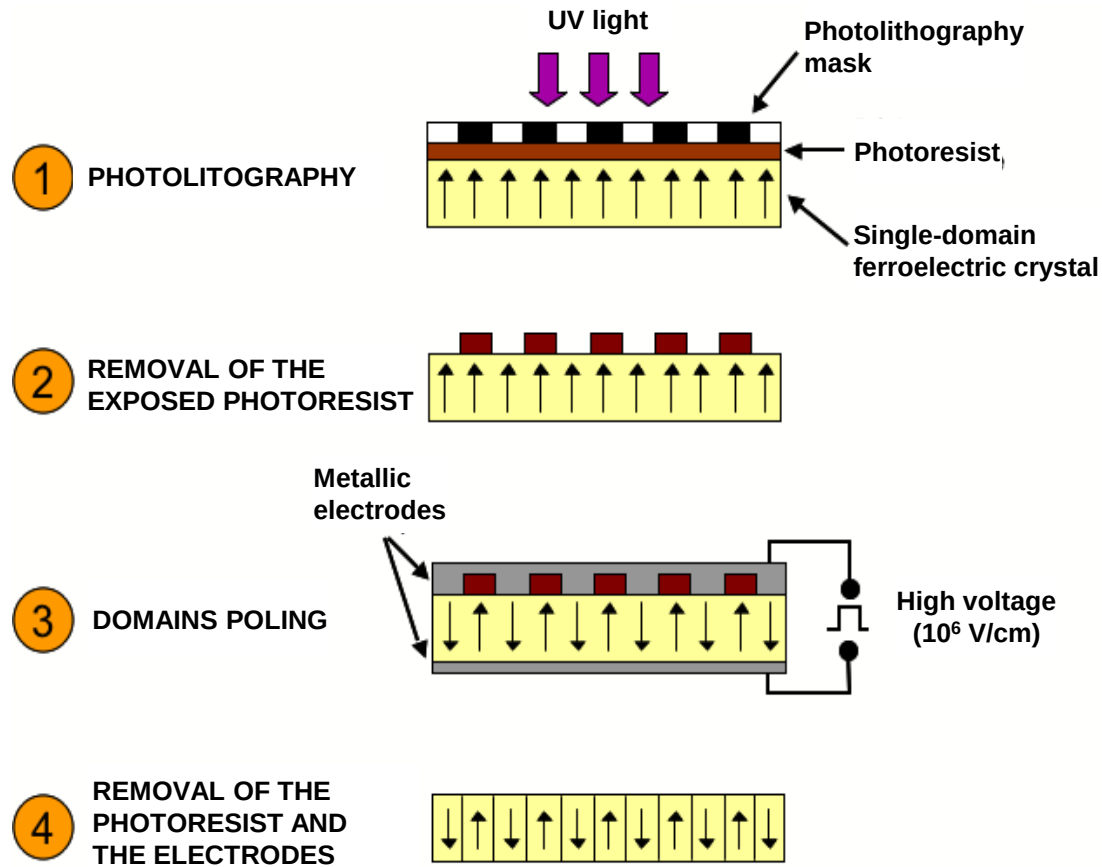
At first glance, it seems less effective than the perfect phase matching.

In practice, it has **many advantages**, including:

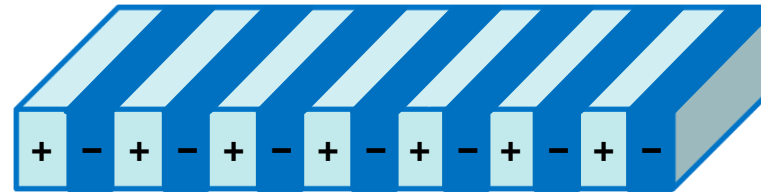
- Solicitation of the largest $\chi^{(2)}$ tensor coefficient (e. g. d_{33})
- Engineering of non-linear interaction
- A configuration without double refraction can always be chosen

Quasi-phase matching techniques

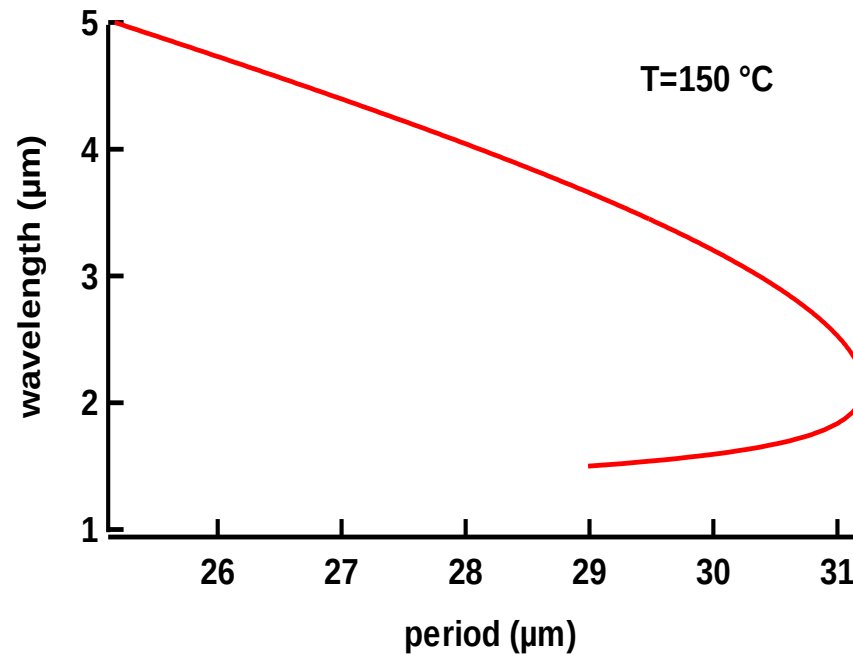
Electric-field poling in ferroelectrics (LiNbO_3 , KTP, LiTaO_3 ,...)



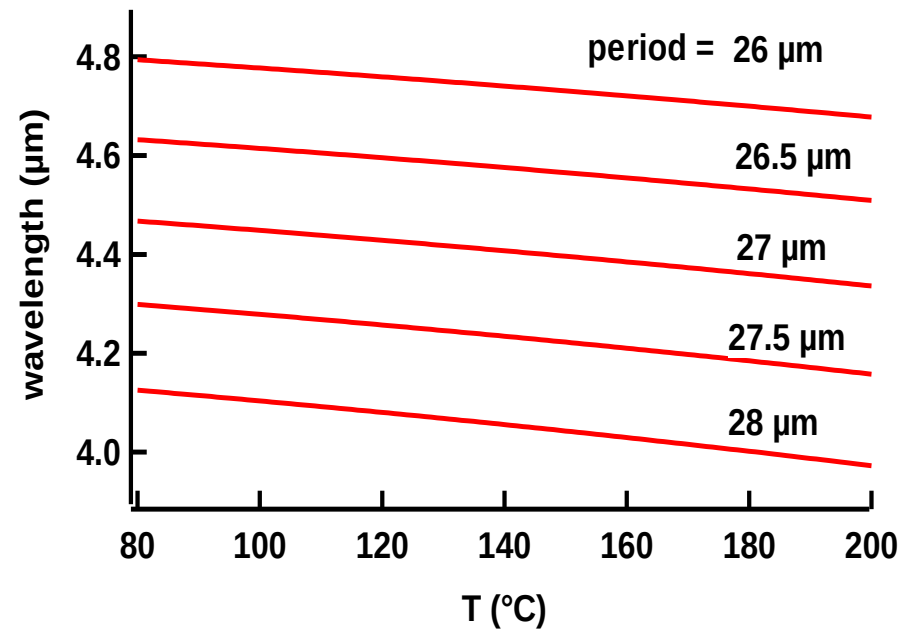
Periodically Poled Lithium Niobate (PPLN)



Periodic quasi-phase matching



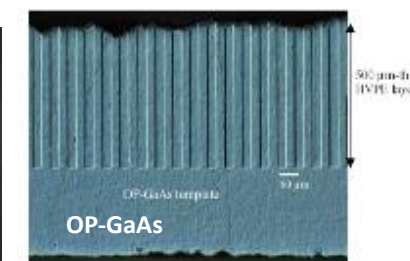
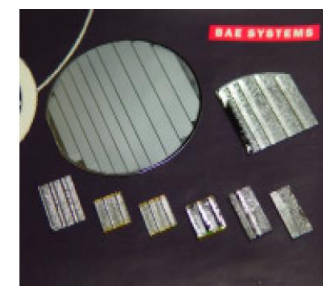
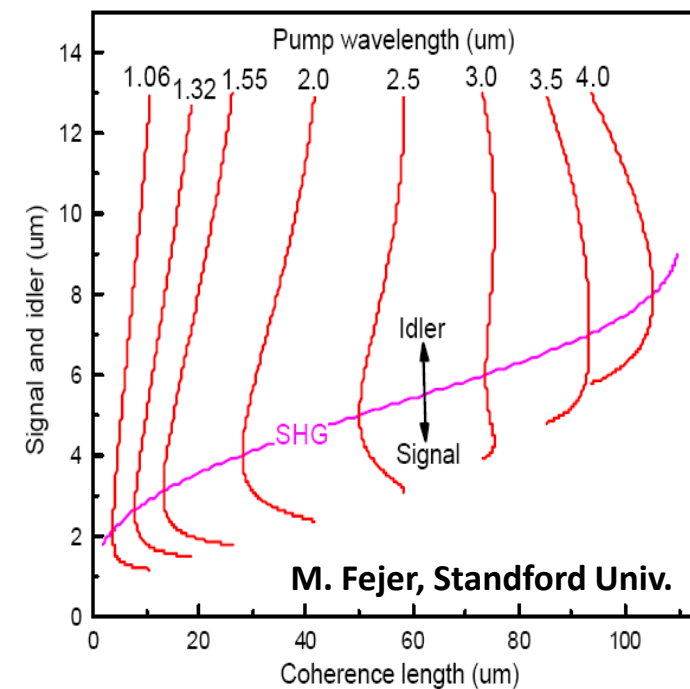
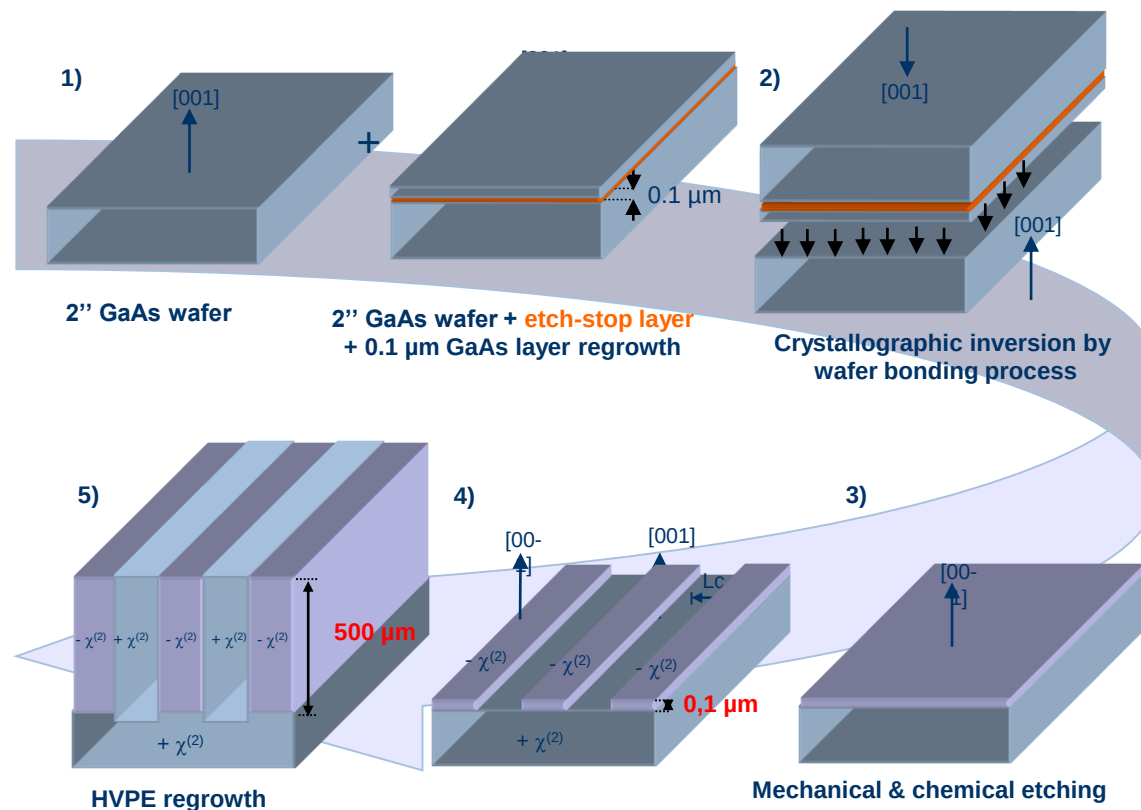
Generated wavelength
for a 1.064 μm pump



Generated wavelength
as a function of temperature

Quasi-phase matching techniques

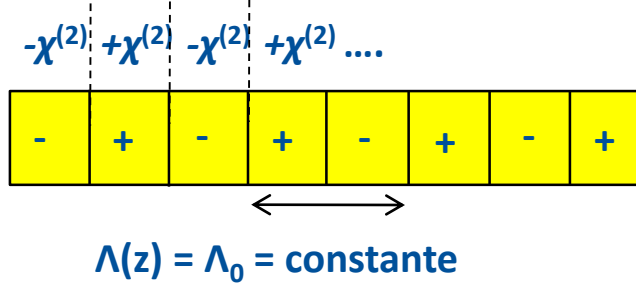
Epitaxial growth (GaAs, GaP, GaN,...)



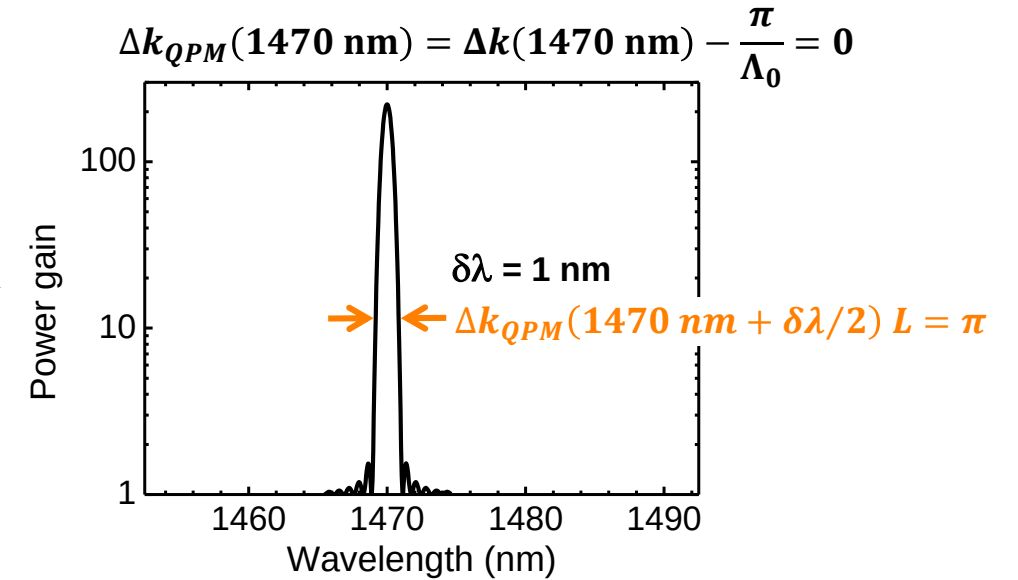
THALES
III-V lab

Aperiodic quasi-phase matching: parametric gain spectrum engineering

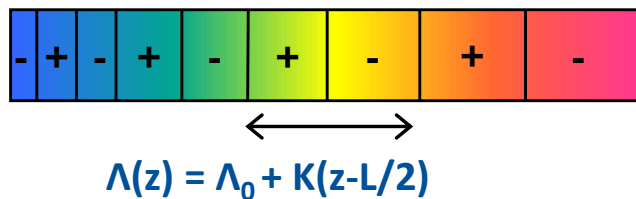
Nonlinear crystal with periodic poling



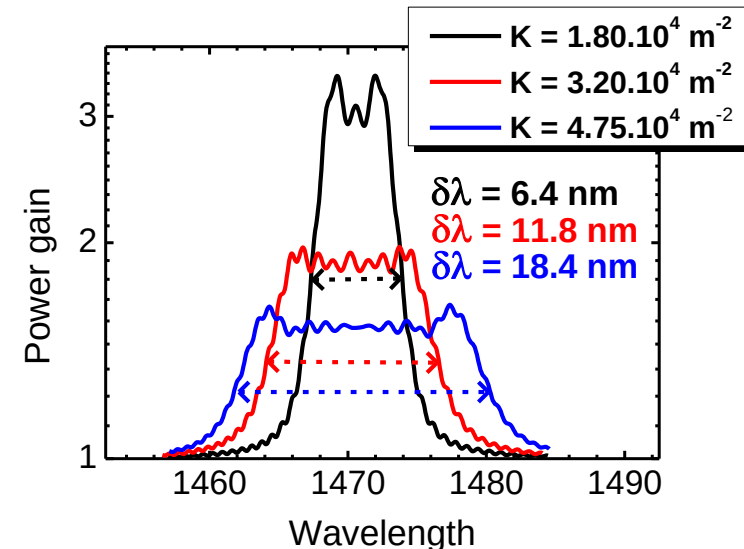
- Central wavelength fixed by the poling period
- Bandwidth limited by crystal length and chromatic dispersion



Nonlinear crystal with chirped period poling



- Engineering of the gain spectrum
- No magic!: gain x bandwidth = cst

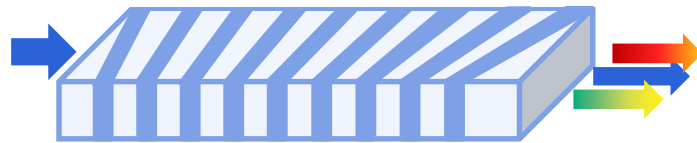


Broadband parametric source

Engineering of quasi-phase matching



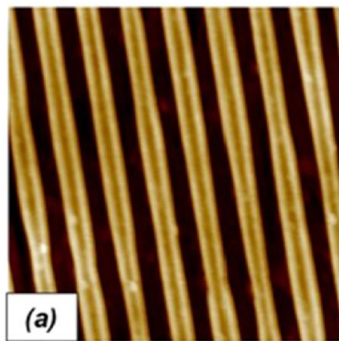
Quasi-phase matching with two periods



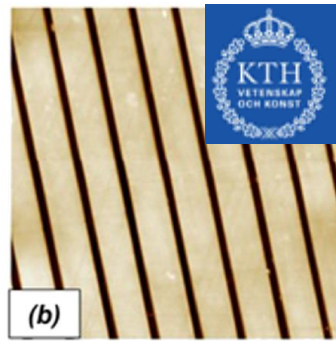
Fan-out quasi-phase matching



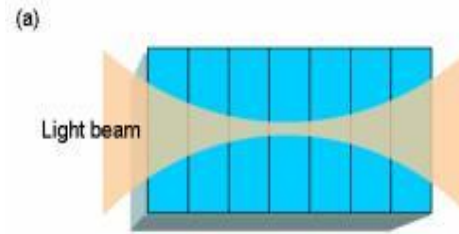
Aperiodic quasi-phase matching



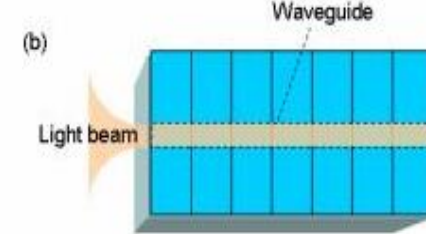
(a)



(b)



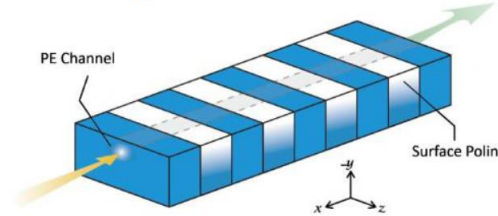
The light beam is diffractive in the bulk chip
The power level depends on the focusing condition and optical properties of the crystal



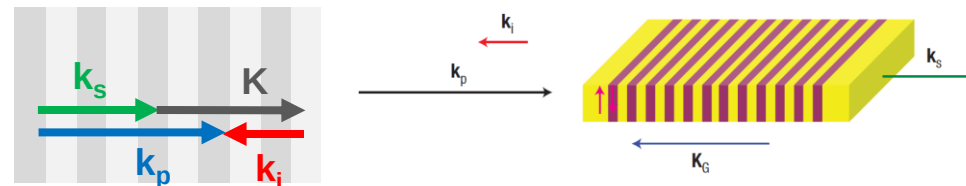
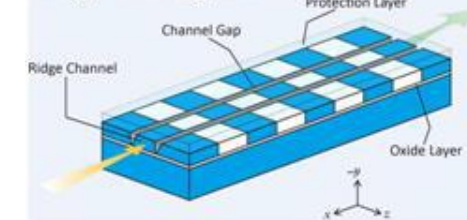
The light beam is confined in the waveguide chip.
The power level is usually less than 1W to avoid damaging the waveguide structure.

source :
HCP
HC PHOTONICS CORP.

RPE waveguide



Ridge Waveguide



Backward-wave quasi-phase matching

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Reminder: three-wave mixing equations

We assume that $\Delta k = k_3 - k_1 - k_2 \approx 0$ for $\omega_3 = \omega_1 + \omega_2$

➡ Processes coupling these three waves are effective

...and we assume the absence of effective coupling processes with other waves

Closed system of three differential equations

$$\begin{array}{ll} \omega_3 - \omega_2 \rightarrow \omega_1 & \frac{d}{dz} A_1 = -i \kappa A_3 A_2^* e^{-i \Delta k z} \\ \omega_3 - \omega_1 \rightarrow \omega_2 & \frac{d}{dz} A_2 = -i \kappa A_3 A_1^* e^{-i \Delta k z} \\ \omega_1 + \omega_2 \rightarrow \omega_3 & \frac{d}{dz} A_3 = -i \kappa A_1 A_2 e^{+i \Delta k z} \end{array}$$

$$\text{with } \kappa = \frac{1}{2} \frac{\chi_2}{c} \sqrt{\frac{\omega_1 \omega_2 \omega_3}{n_1 n_2 n_3}} \quad \text{and} \quad A_j = \sqrt{\frac{n_j}{\omega_j}} E_j$$

Parametric amplification

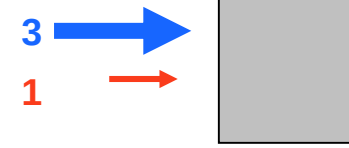
Undepleted pump approximation
with perfect phase matching

$$A_3(z) \approx A_3(0) \sim \text{cste}$$

with parametric gain

$$\frac{d}{dz} A_1 = -i g A_2^*$$

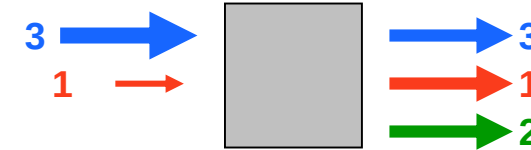
$$\frac{d}{dz} A_2 = -i g A_1^*$$



$$g = \frac{1}{2} \frac{\chi_2}{c} \sqrt{\frac{\omega_1 \omega_2}{n_1 n_2}} E_3(0)$$

$$\begin{aligned} A_1(z) &= A_1(0) \cosh(gz) \\ A_2(z) &= -i A_1(0)^* \sinh(gz) \end{aligned}$$

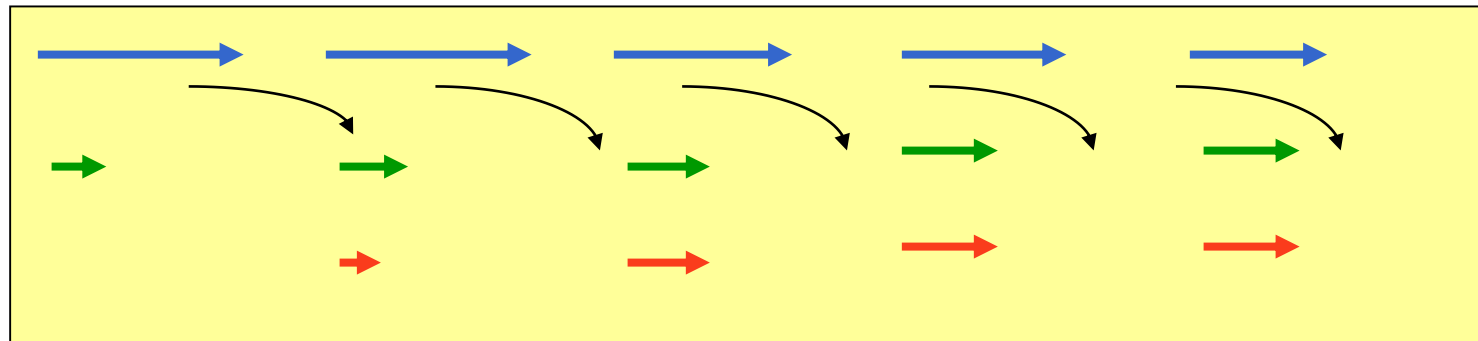
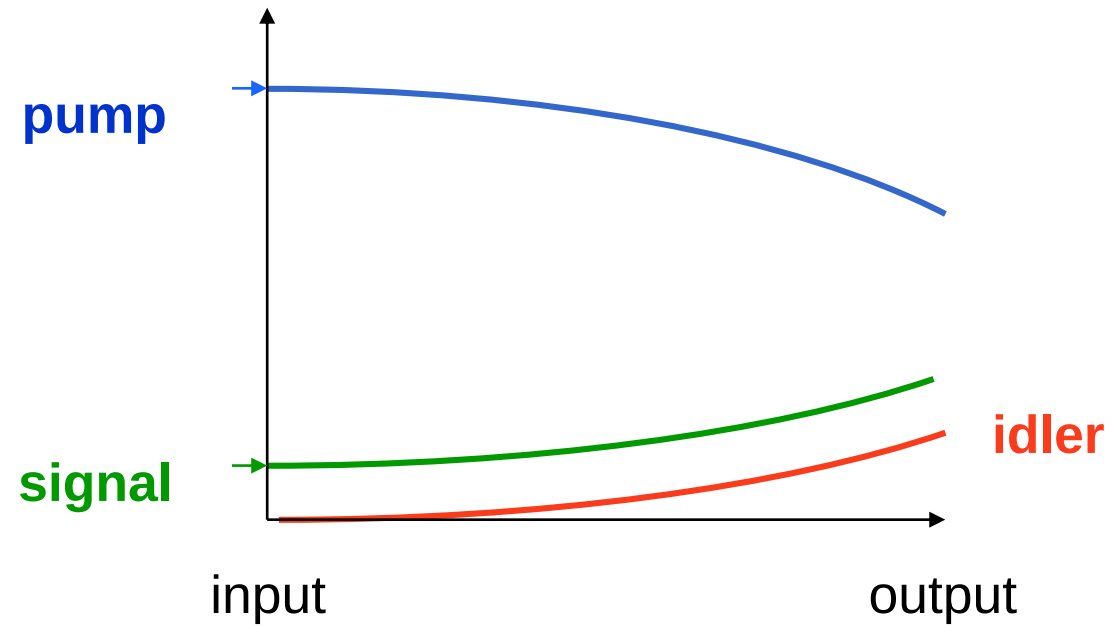
for $A_2(0) = 0$



Amplification of signal wave A_1 and generation of idler wave A_2 (and vice-versa)

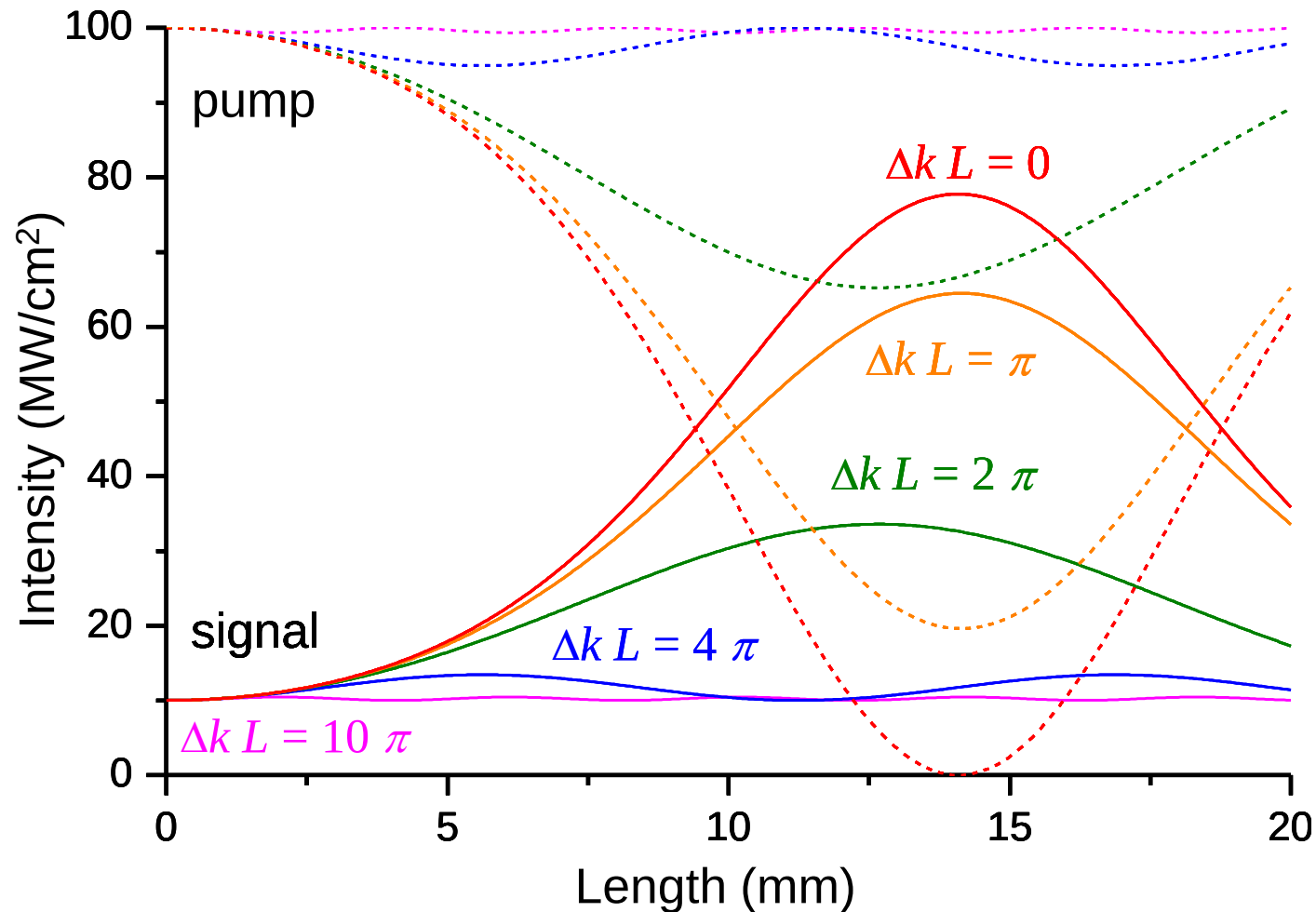
For high parametric gain $A_1(z) \approx A_2(z) \approx \frac{1}{2} A_1(0) e^{gz}$

Parametric amplification and accompanying DFG

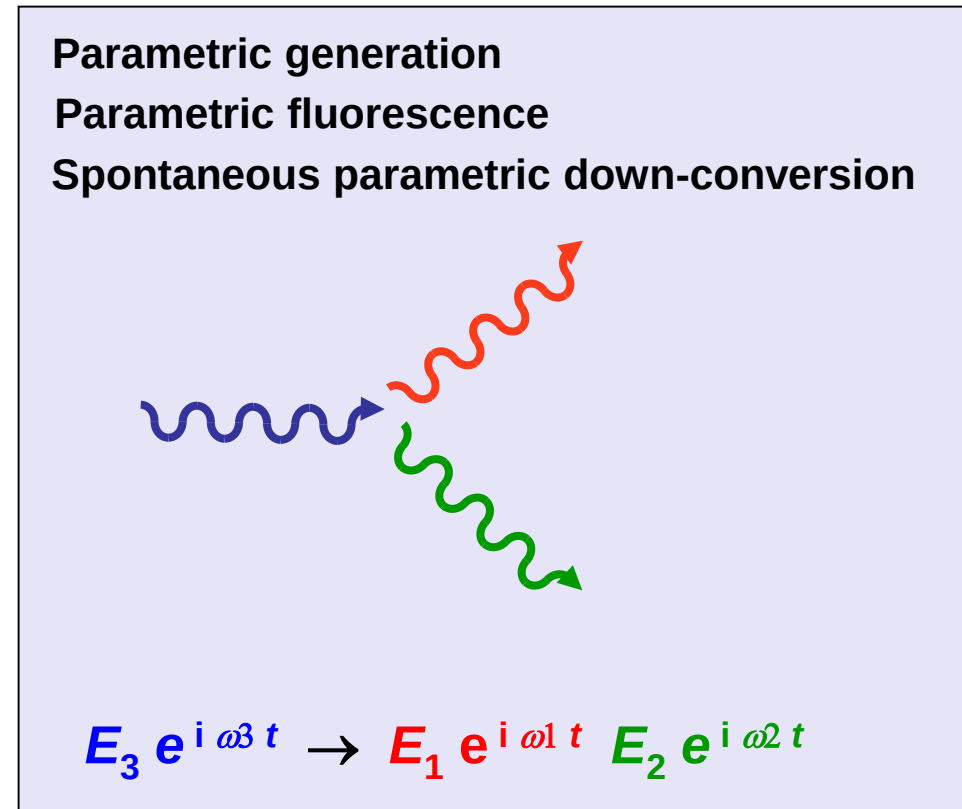
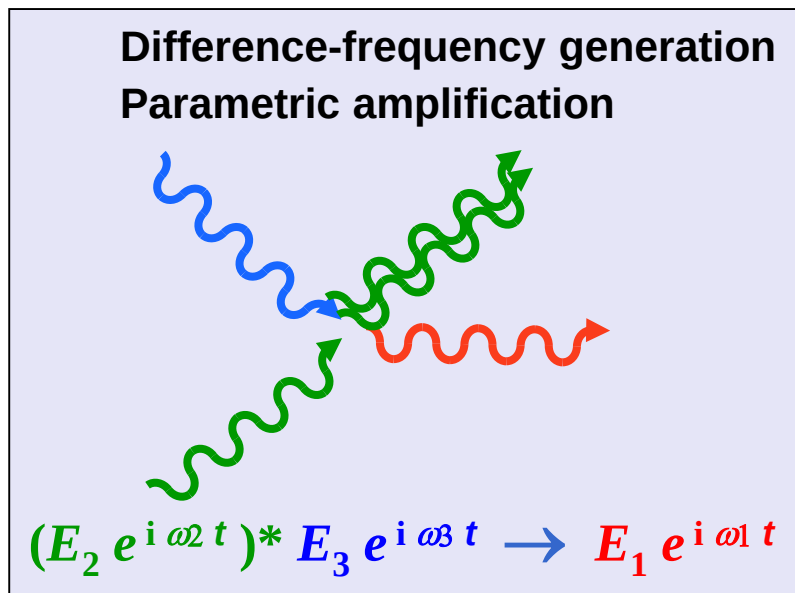
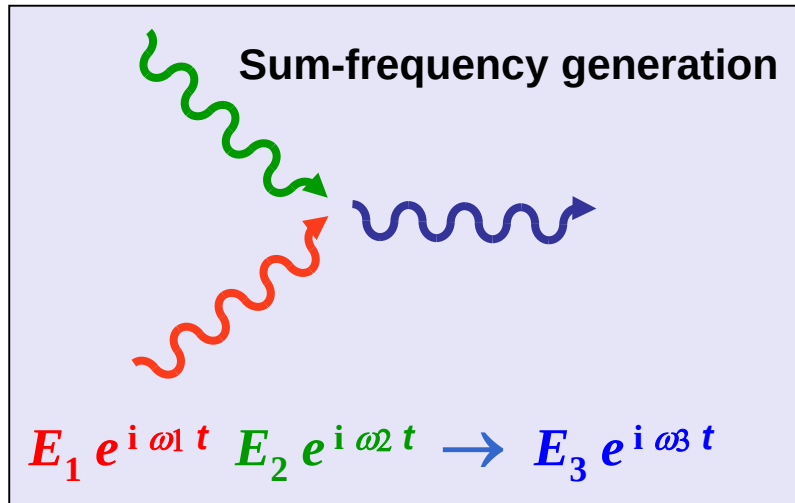


Saturation of parametric gain and pump reconversion

The analytical solution exists, but it is complicated and involves Jacobi's elliptic functions
In the crystal: energy exchange between pump and signal (and idler)

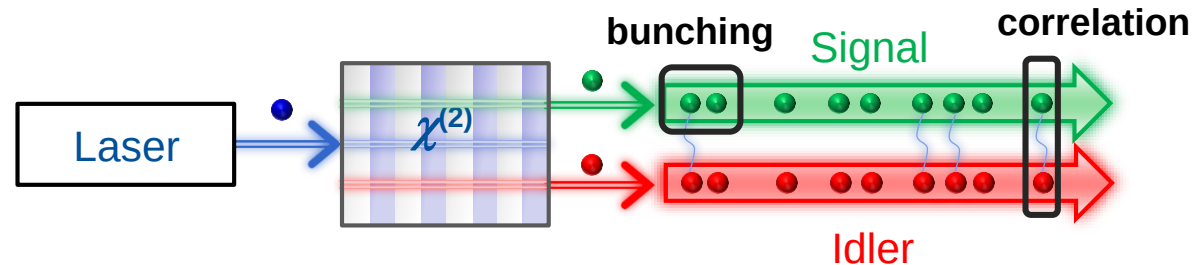


Sum-, difference-frequency generation and parametric fluorescence



Spontaneous parametric down-conversion (SPDC)

Intrinsically quantum phenomenon



➡ Generation of Fock states, of entangled photon pairs

$$|\psi\rangle = \frac{1}{\sqrt{G}} \sum_{n=0}^{\infty} \left(1 - \frac{1}{G}\right)^{n/2} |n_s, n_i\rangle$$

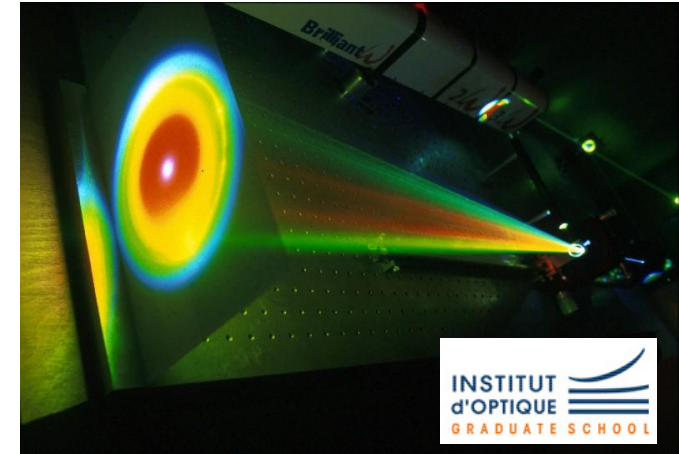
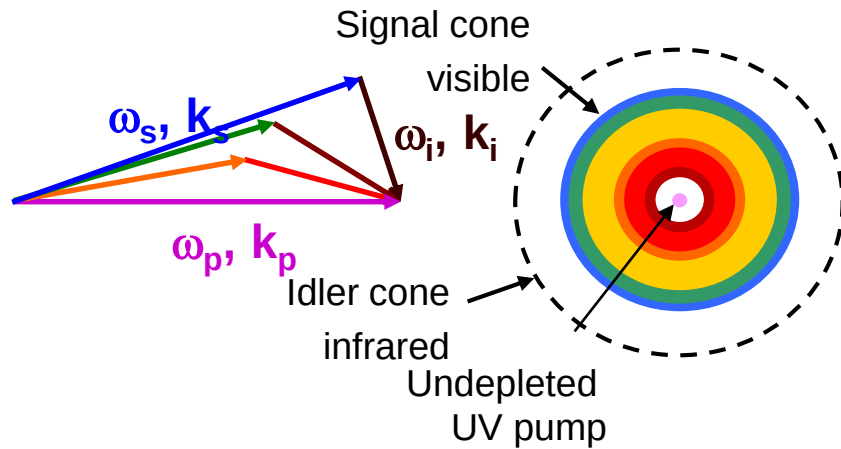
used in numerous experiments and applications in quantum optics (quantum cryptography, quantum information, etc.)

- For $G \approx 1$, pairs are generated spontaneously and their amplification in the rest of the crystal can be neglected → this is referred to as SPDC; this case concerns scientific studies in quantum optics
- Pour $G \gg 1$, pairs are generated and then amplified (~avalanche effect) with potentially high efficiencies → this is referred to as OPG; this case may be of interest in quantum optics, but also in classical optics

Optical parametric generation and fluorescence

Fluorescence cone

Non-collinear interactions



Each direction of the cone corresponds to a pair (ω_s, ω_i) such that (k_s, k_i) approximately verify the non-collinear phase matching with fixed k_p .

→ Generation of two superimposed light cones, signal cone and complementary cone.

Typical gain in continuous-wave regime

Example: lithium niobate LiNbO_3

$$\chi_2 = \frac{4}{\pi} 27 \text{ pm/V}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$n \approx 2.13$$

$$I_p = \frac{P_p}{\pi w_p^2/2}$$

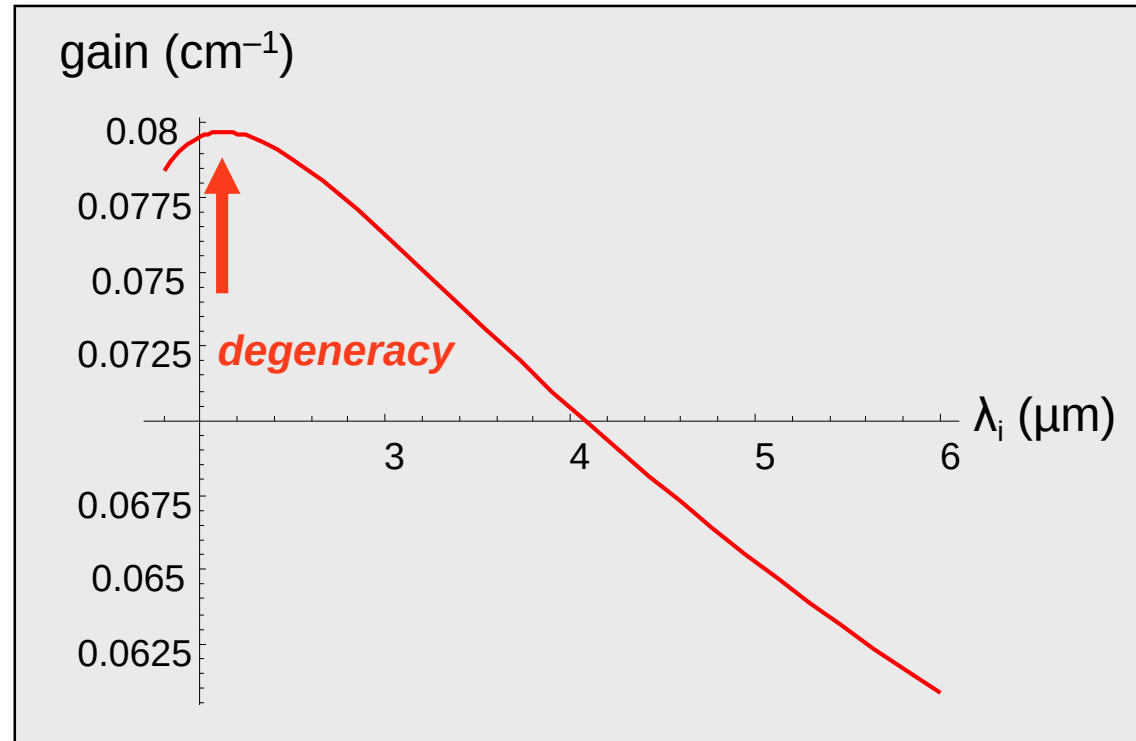
$$w_p = 45 \text{ } \mu\text{m}$$

$$P_p = 1 \text{ W}$$

$$\lambda_p = 1.06 \text{ } \mu\text{m}$$



$$g = \chi_2 \sqrt{\frac{2\pi^2 Z_0}{n^3 \lambda_i \lambda_s}} I_p$$



Not many photons coming out of the OPG... what to do?

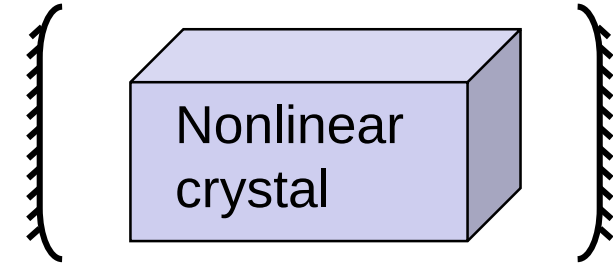
Optical parametric oscillator

Parametric interaction = process with gain for signal and complementary

However, in most cases (except for short pulses), the single-pass gain is quite low

- ➡ The nonlinear medium is inserted in an optical cavity to obtain positive feedback and realize an oscillator
Oscillation builds up from quantum noise and high efficient can be achieved

➡ **Optical parametric oscillator (OPO)**

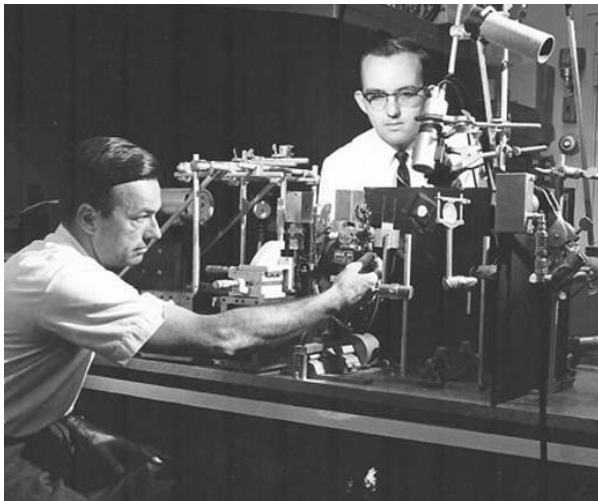
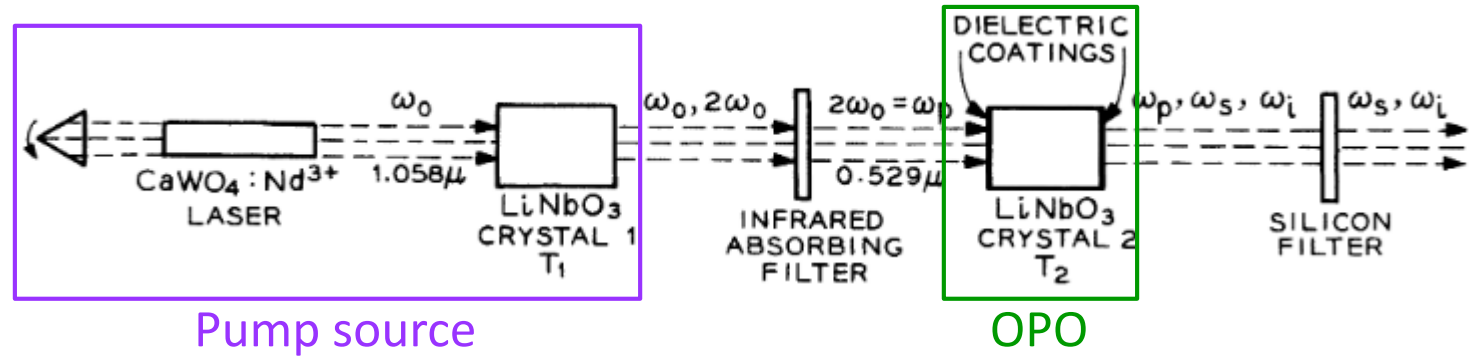


OPO = Optical synthesizer:

- ability to emit coherent radiation in spectral ranges where no laser exists
- potential to tune the emitted wavelengths over very wide ranges (several micrometers), much wider than with the most tunable lasers

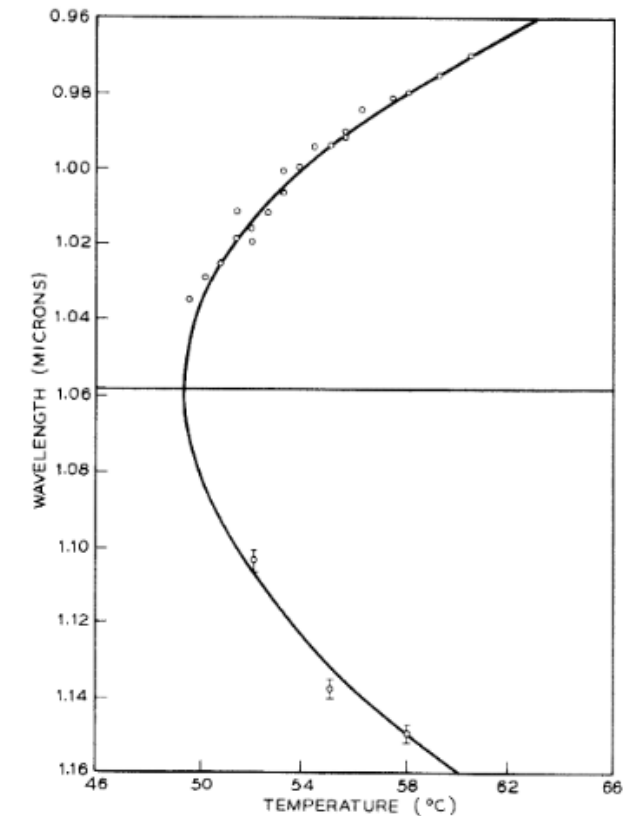
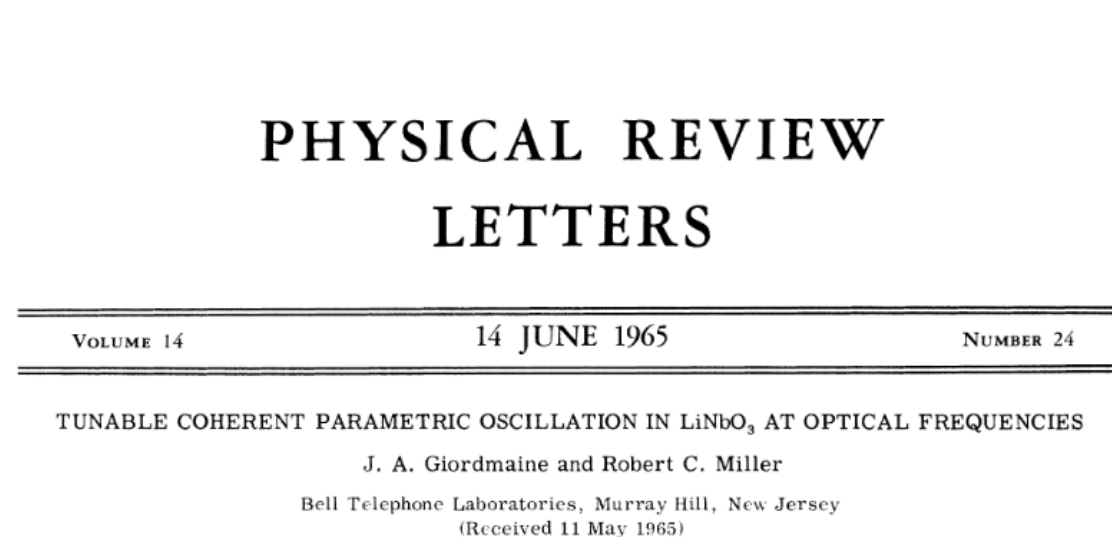
First OPO

Giordmaine & Miller (1965)



J. A. Giordmaine & R. C. Miller

Source: Optica

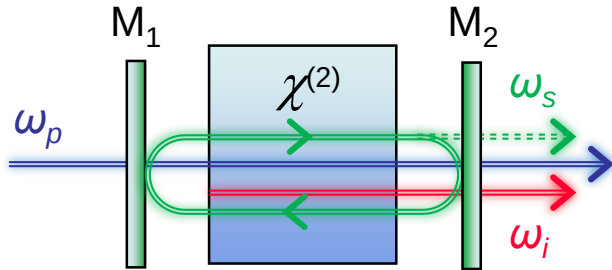


Optical parametric oscillators

Three interacting waves → greater variety of resonator configurations than with lasers

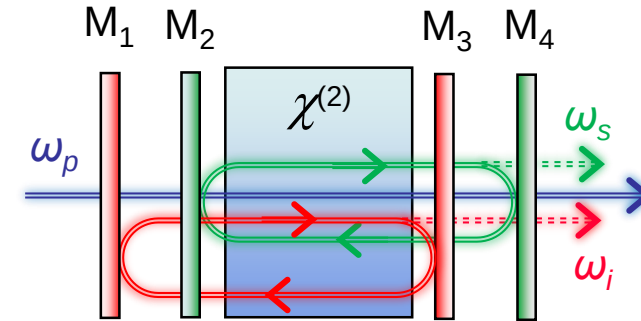
Non-exhaustive list:

Singly resonant OPO



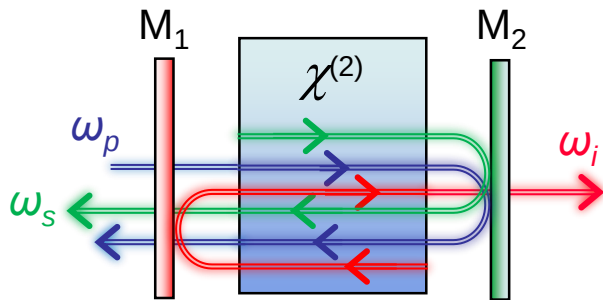
Simple, rather high threshold (~few W in CW regime)

Doubly resonant OPO



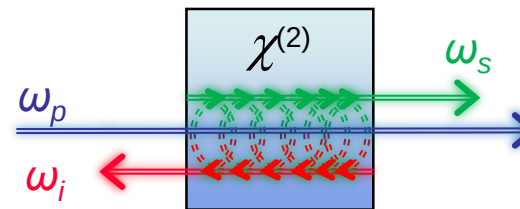
More complex, low threshold (~few 10 mW to few 100 mW in CW regime)

Cross-resonant OPO



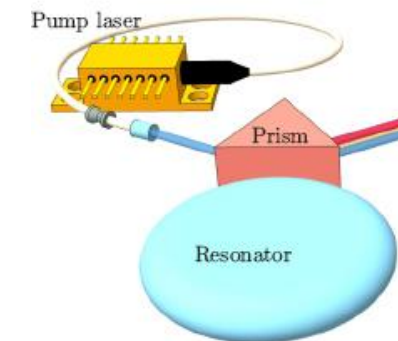
Less de saturation, high threshold (~few 100 W → pulsed regime)

Mirrorless backward-wave OPO



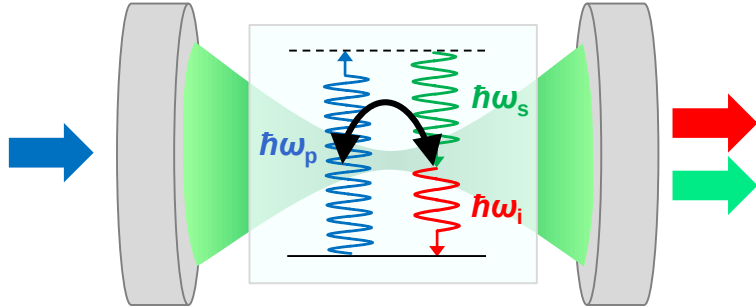
Super simple, high threshold (~few kW → pulsed regime)

Gallery-mode monolithic OPO



Triply resonant, very low threshold (< 1 mW in CW regime)

Optical parametric oscillator: Tuning of emitted frequencies



=

Modification of the phase matching condition:

- temperature
- orientation
- quasi-phase matching nonlinear grating period
- ...

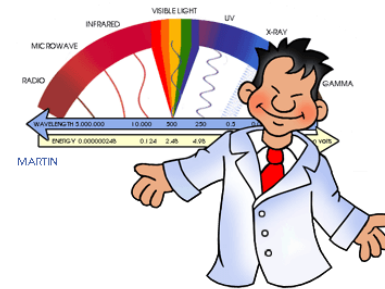
Conservation of energy

$$\omega_p = \omega_s + \omega_i$$



Phase matching

$$k_p = k_s + k_i (+ k_\Lambda)$$

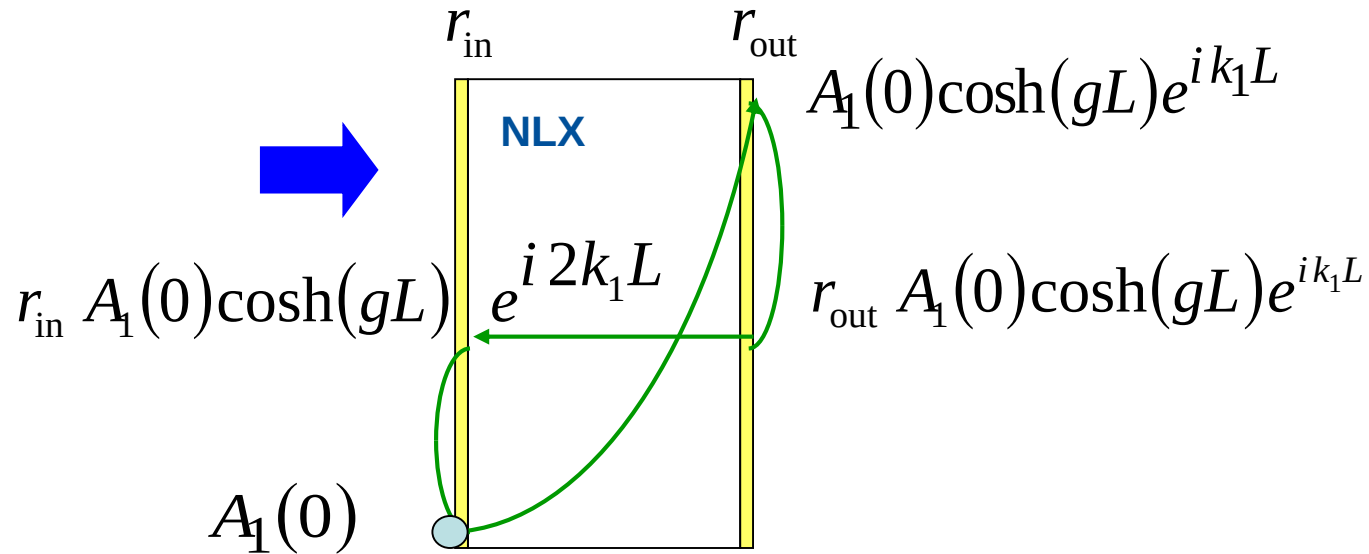


1.064 μm

1.4 $\rightarrow$ 1.6 μm

3 $\rightarrow$ 5 μm

Singly resonant OPO



$$A_1(0) = r_{in} r_{out} \cosh(gL) e^{i2k_1 L} A_1(0)$$

$$\cosh(g_{th} L) = \frac{1}{\sqrt{R_{in} R_{out}}}$$

$$k_1 L = m \pi$$

Condition on the pump amplitude

Resonance condition

Example: lithium niobate LiNbO_3

$$\chi_2 = \frac{4}{\pi} 27 \text{ pm/V}$$

$$c = 3 \times 10^8 \text{ m/s}$$

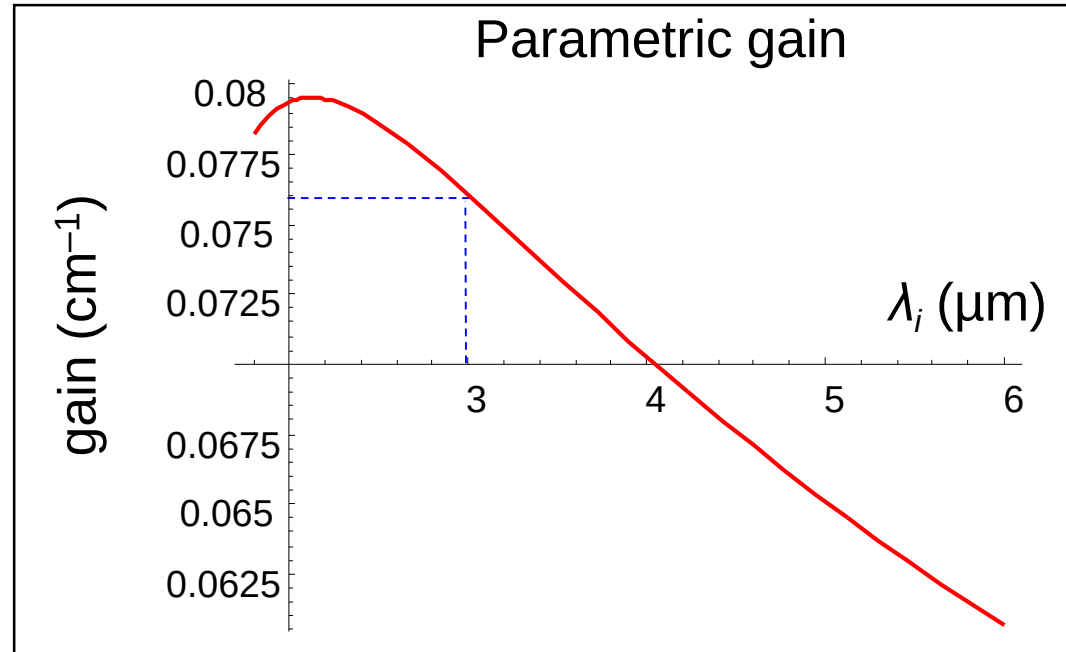
$$n \approx 2.13$$

$$I_p = \frac{P_p}{\pi w_p^2/2}$$

$$w_p = 45 \text{ } \mu\text{m}$$

$$\lambda_p = 1.06 \text{ } \mu\text{m}$$

$$g_s L \approx \sqrt{T_e + T_s}$$



Seuil à 3 μm

$$R_{\text{in}} = R_{\text{out}} = 0.99$$

$$L = 1 \text{ cm}$$

$$\left. \begin{aligned} g &= 0.076 \sqrt{P(\text{W})} \text{ cm}^{-1} \\ \Rightarrow g_{\text{th}} &= 0.14 \text{ cm}^{-1} \end{aligned} \right\} \Rightarrow P_s \approx 3.4 \text{ W}$$

Nonlinear parametric optics

1. Principles of three-wave mixing

- Nonlinear response
- Difference-frequency generation: conversion efficiency and acceptance
- Phase- and quasi-phase-matching

2. Parametric amplification, generation, and oscillation

- Parametric amplification and reconversion effects
- Parametric fluorescence
- Parametric oscillation, variations of OPO schemes

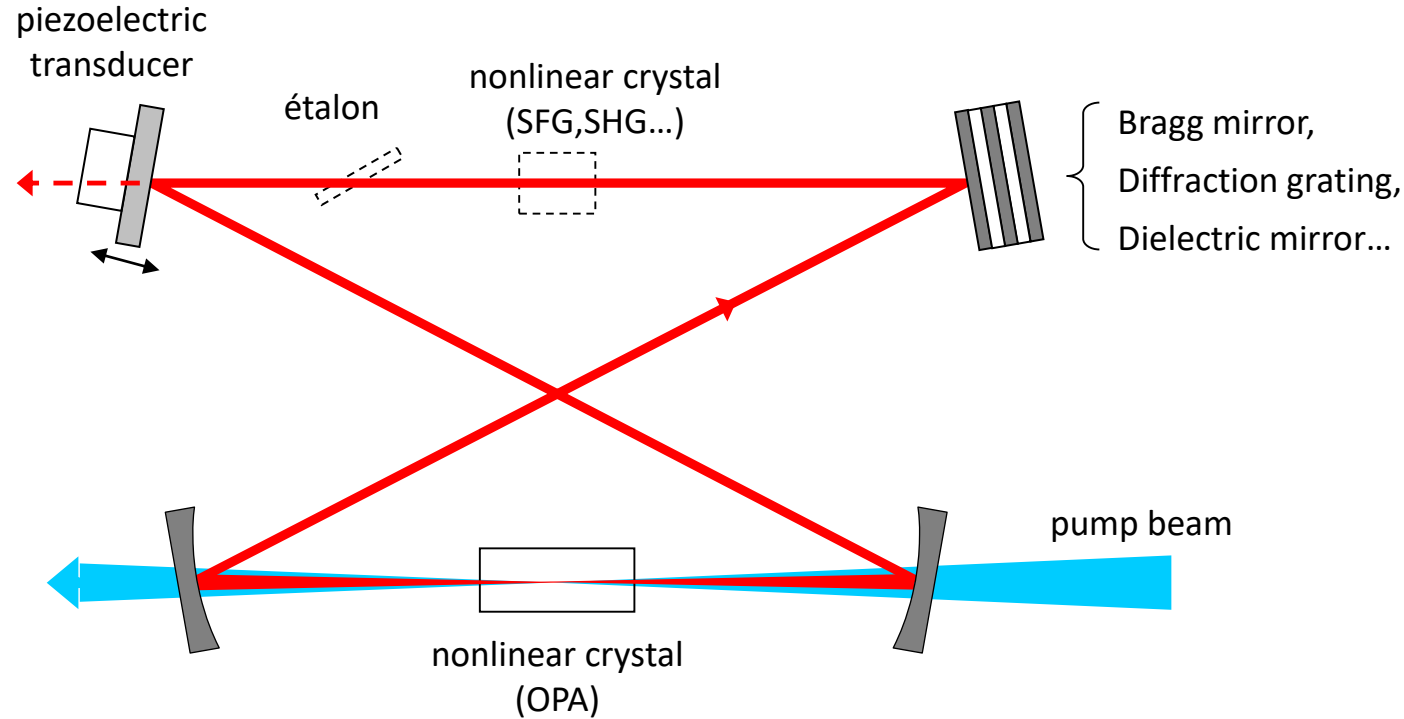
3. Implementations and related applications

- Continuous-wave regime
- Nanosecond regime, spectral content issues
- Picosecond to femtosecond regime

Conclusion

OPOs in continuous-wave (CW) regime

Typical architecture

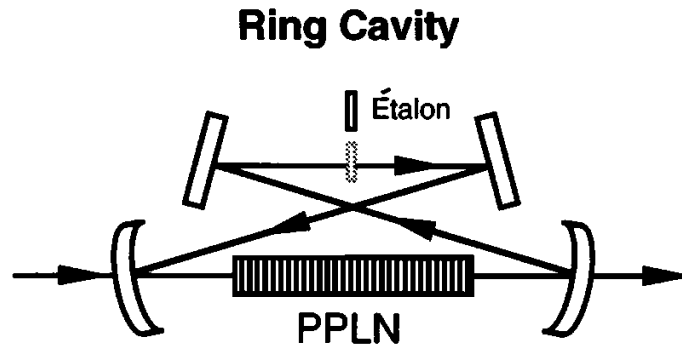


The issue of the low gain issue in CW regime

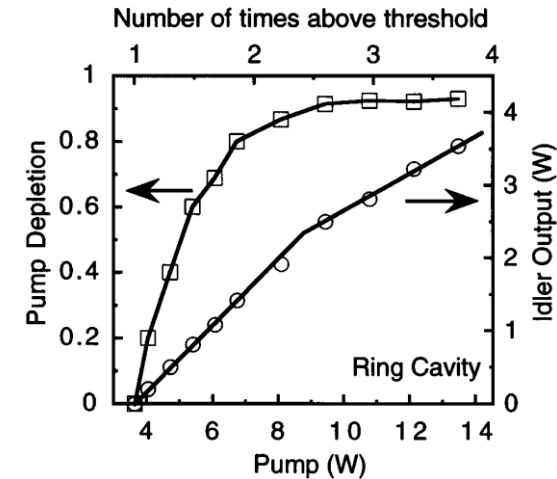
- Highly focused beams in the nonlinear crystal (typical diameter $\sim 50 \mu\text{m}$)
- Stable optical resonator allowing good overlap between the three waves and minimum diffraction losses
- High mirror reflection coefficients for the resonant wave

CW OPOs

Very high conversion efficiencies are achievable

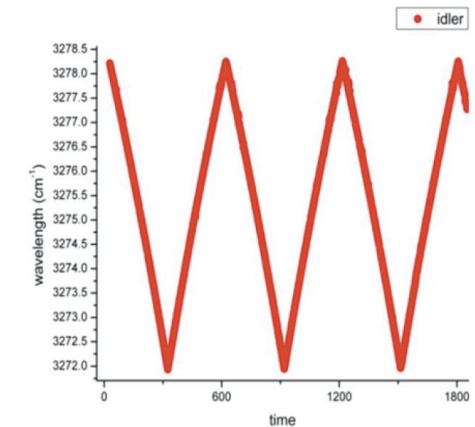


Bosenberg *et al*, Opt. Lett. 21 (1996) 1336



Main application fields related to spectrometry

- High-resolution spectroscopy
- Gas analysis
- Material characterisation
- Chemical reactions
- Calibration of infrared detectors
- Microscopy for biotechnology
- Quantum technologies



Arslanov *et al*, Laser Photon. Rev. 21 (2012) 1-19
Radboud University (The Netherlands)

Photonic integrated circuit (PIC) optical parametric oscillators

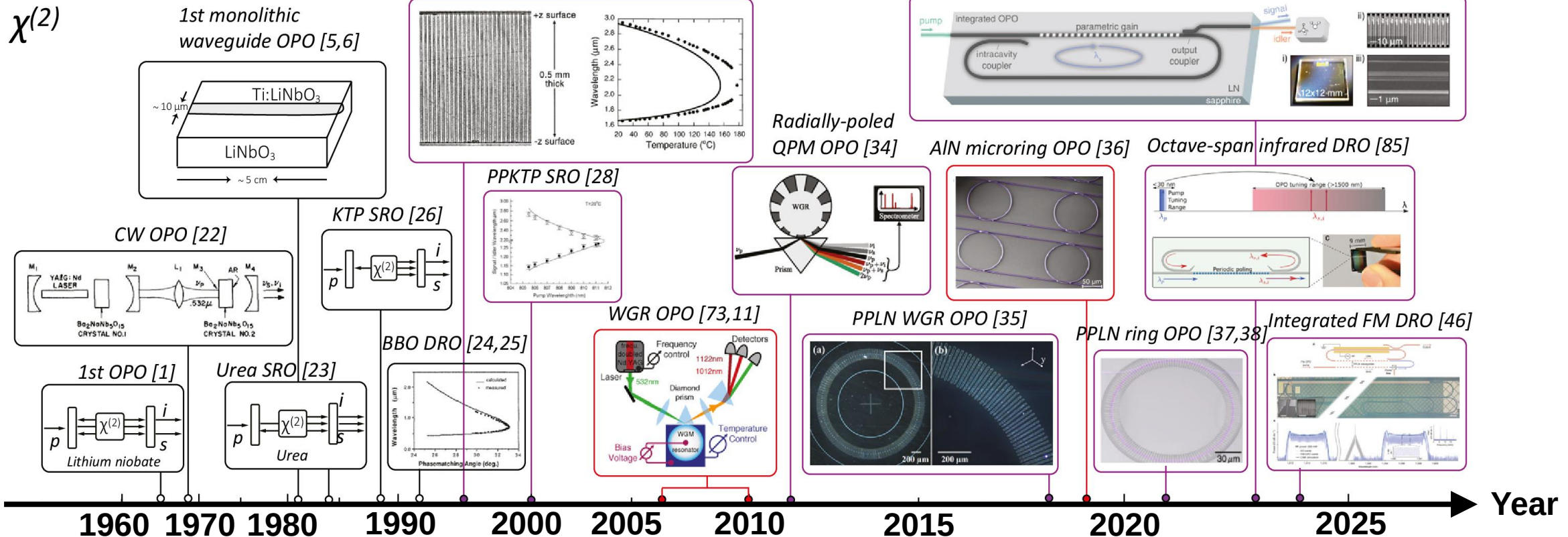
Toward on-chip OPO's in a PIC platform

Lu et al, Optica 13, 11-30 (2026)

Lasers, nonlinear crystals, and platform development

On-chip miniaturization with milli-/micro-meter cavities

Integration towards applications in spectroscopy, sensing, etc.



Nonlinear parametric optics

1. Principles of three-wave mixing

- Nonlinear response
- Difference-frequency generation: conversion efficiency and acceptance
- Phase- and quasi-phase-matching

2. Parametric amplification, generation, and oscillation

- Parametric amplification and reconversion effects
- Parametric fluorescence
- Parametric oscillation, variations of OPO schemes

3. Implementations and related applications

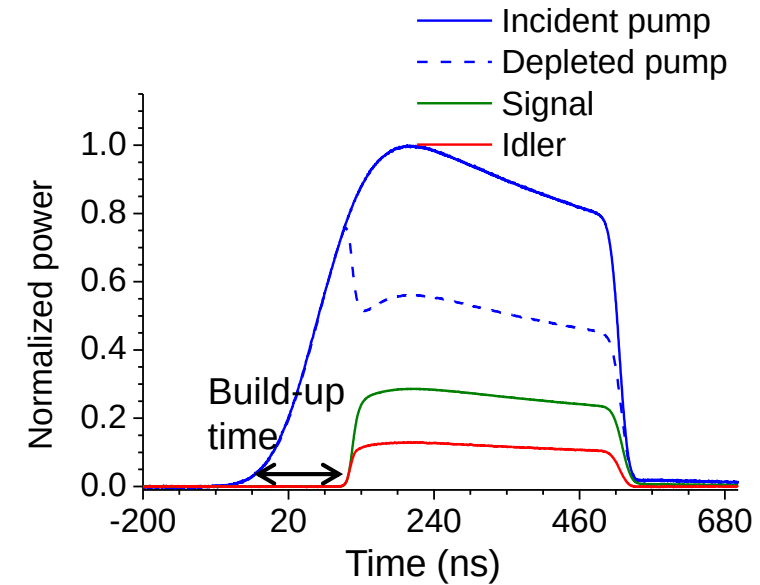
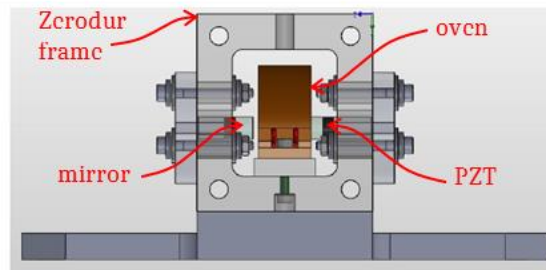
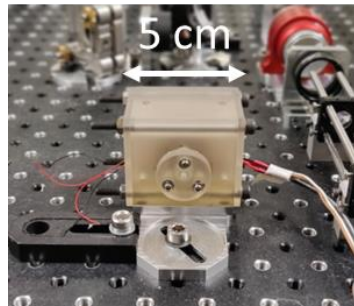
- Continuous-wave regime
- Nanosecond regime, spectral content issues
- Picosecond to femtosecond regime

Conclusion

OPO in nanosecond pulsed regime

Preferred operating regime for OPOs

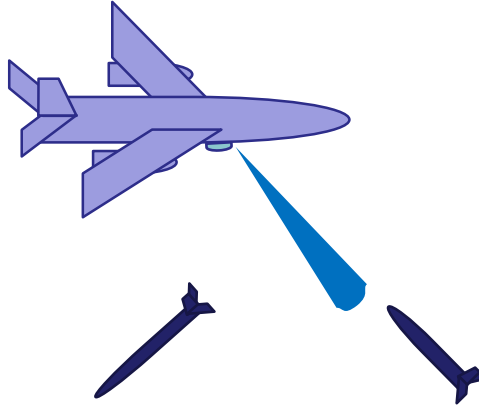
- The high peak power of Q-switched lasers allows to reach high gains
→ the constrain on cavity loss is highly relaxed
- Capability to implement simple and compact resonators: a crystal + 2 mirrors



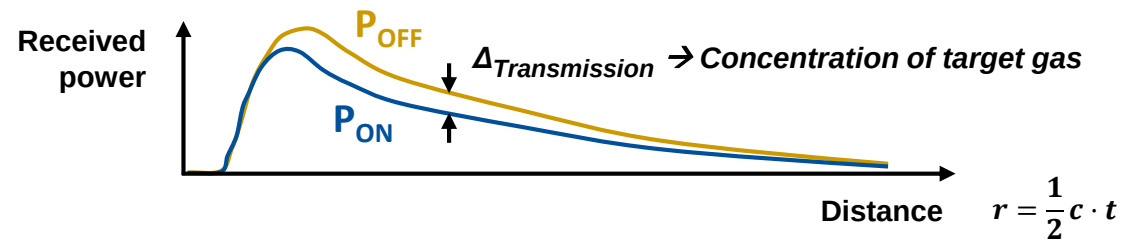
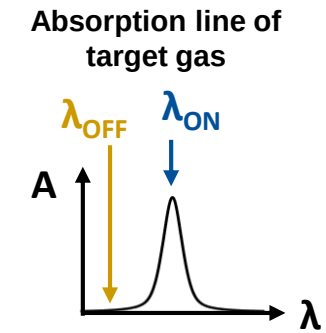
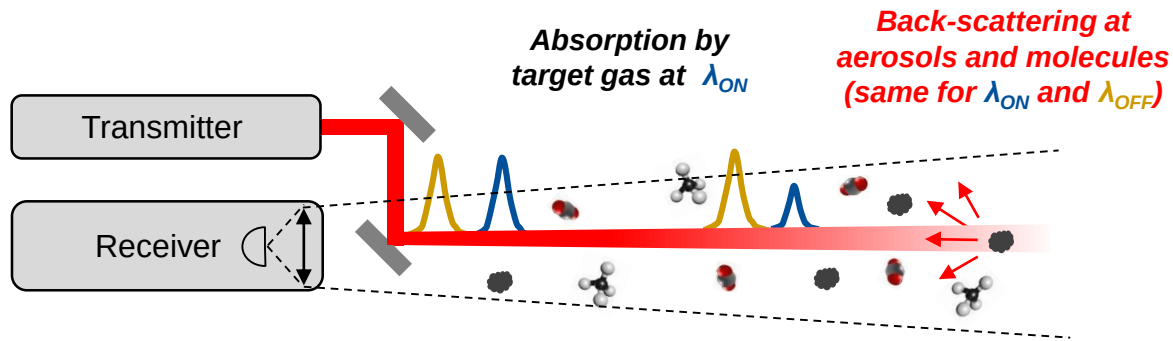
OPO in nanosecond regime

Some applications

- Optical counter measures
- Gas detection (differential absorption lidar)



Infrared signature of an aircraft



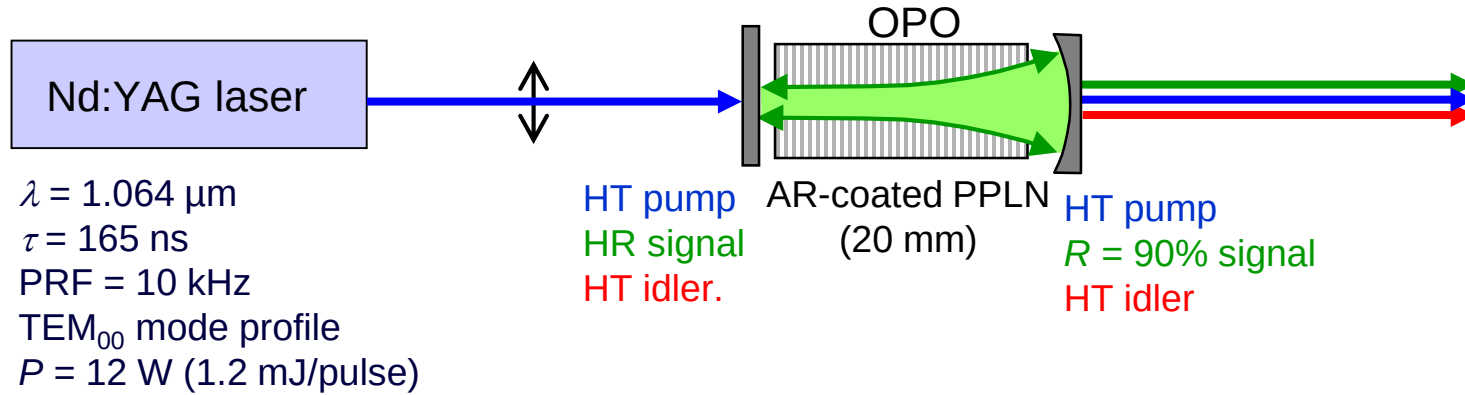
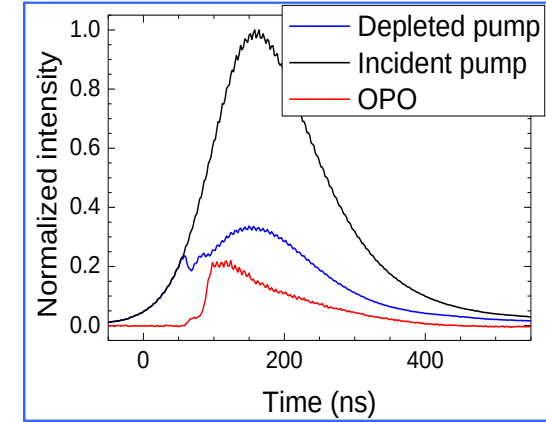
Measurement of gas concentration with spatial resolution along line of sight

OPO in nanosecond regime

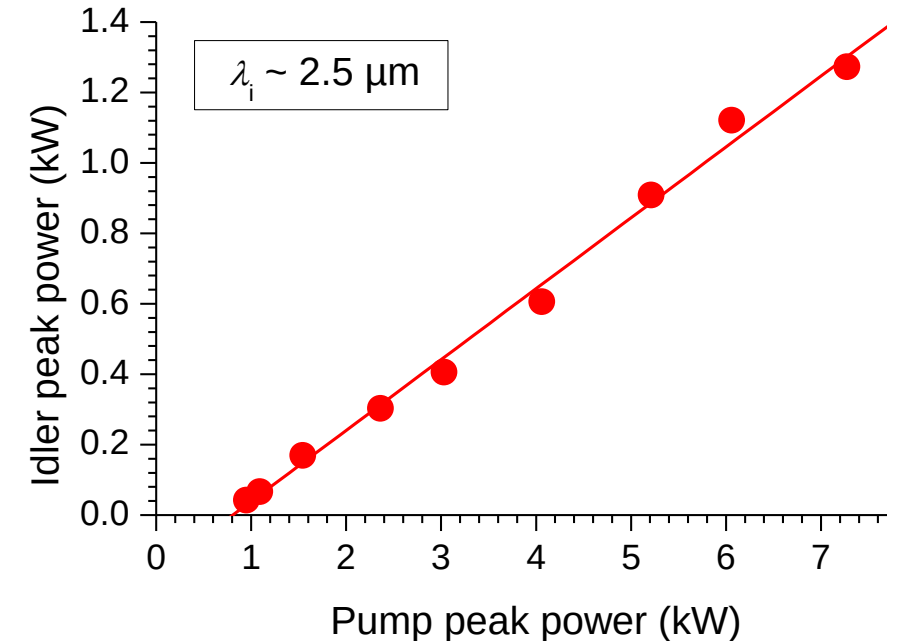
Pulsed oscillation threshold

Case of a singly resonant OPO pumped by a Nd:YAG Q-switched laser

- Pump peak power up to 7 kW
- OPO CW threshold estimated to about 200 W
- Measured pulsed threshold: 800 W peak power



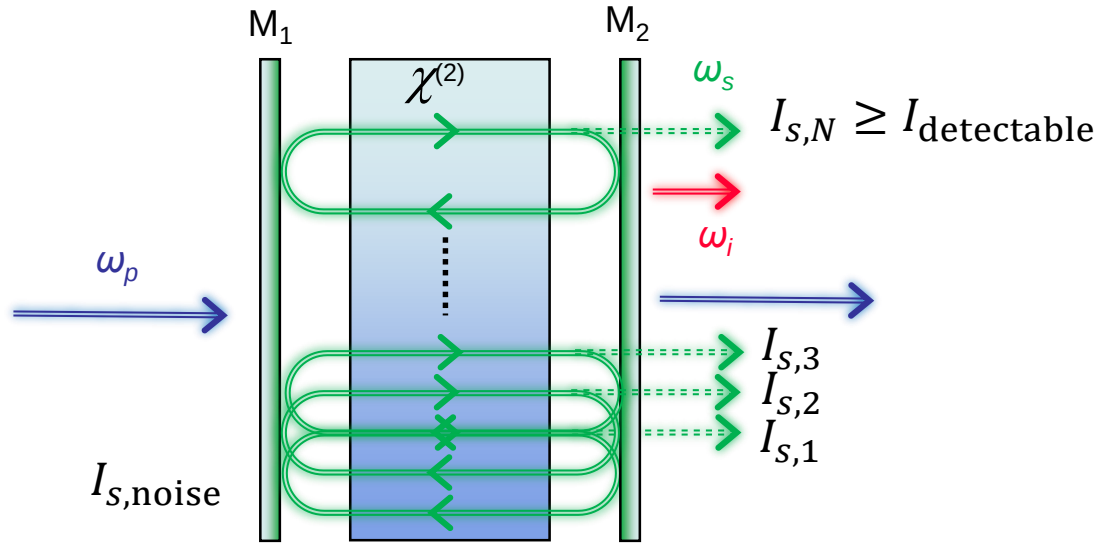
Why?



Oscillation build-up time

Singly resonant OPO

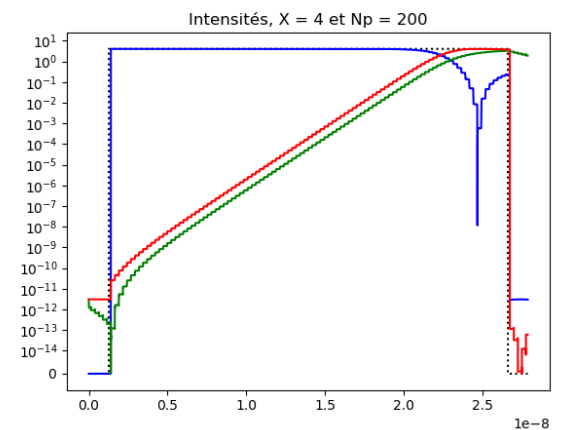
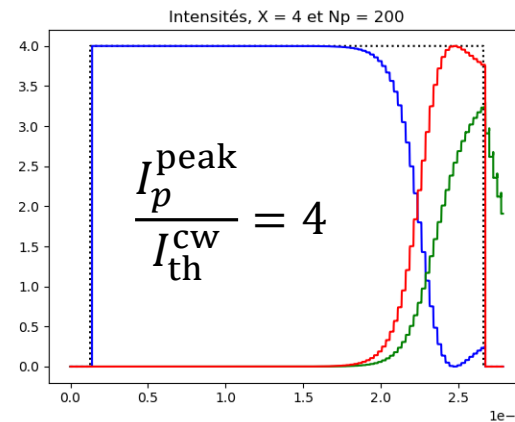
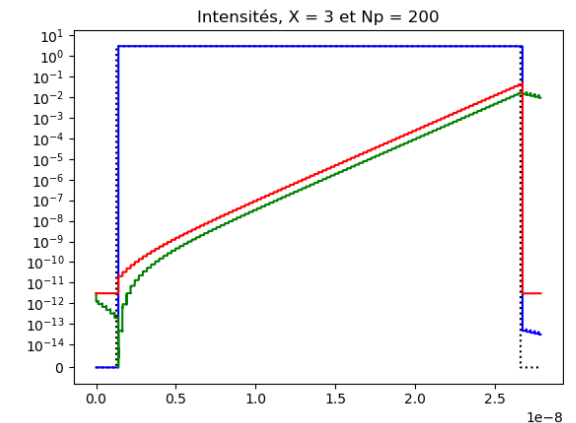
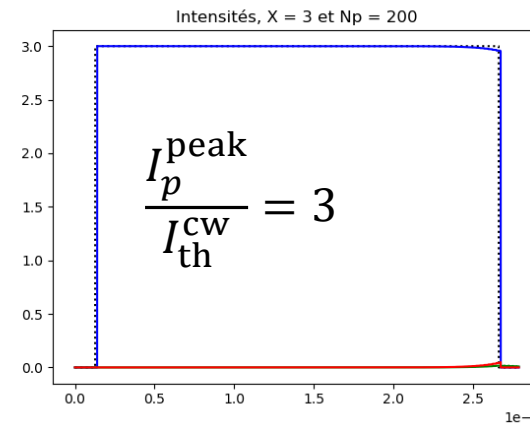
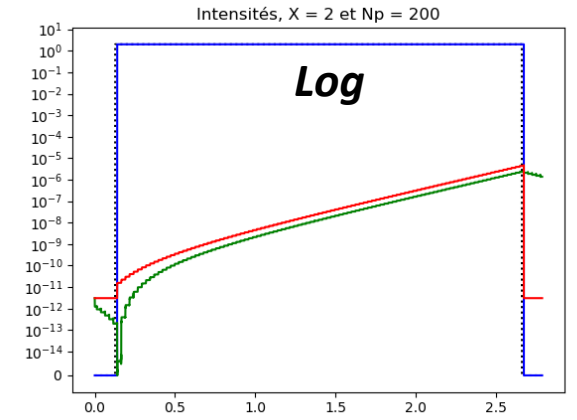
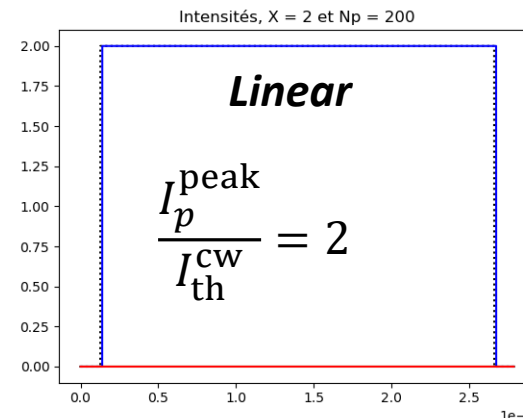
- Build-up time = number of cavity round-trips to reach a detectable signal
- A somewhat arbitrary definition...



$$I_{s,N} = I_{s,noise} [R \cosh^2(\gamma L)]^N$$

$$t_{\text{build-up}}^{\text{SRO}} = N \times t_{\text{round-trip}}$$

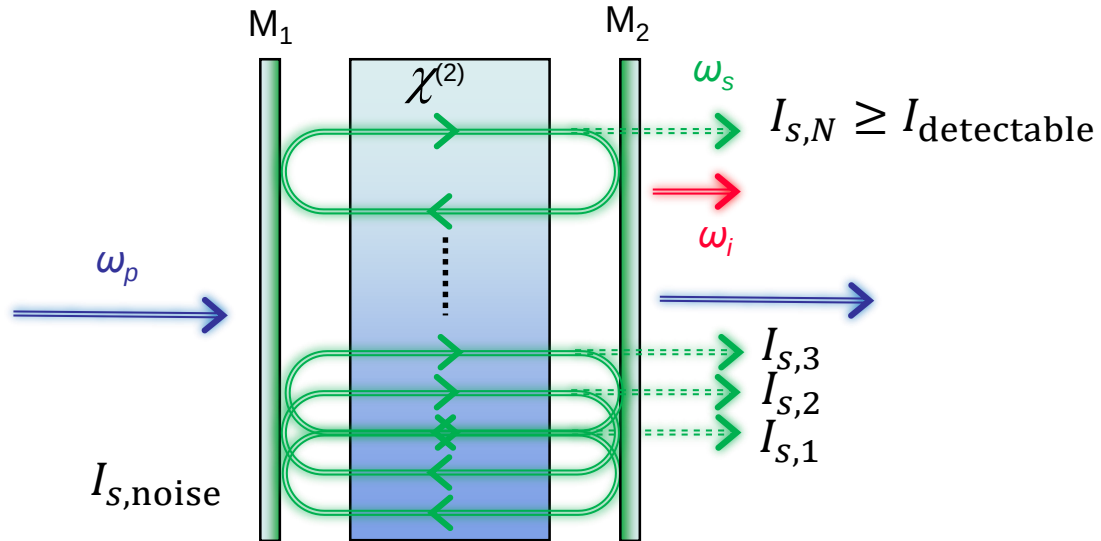
Brosnan & Byer, IEEE J. Quantum Elec. 15 (1979) 415



Oscillation build-up time

Singly resonant OPO

- Build-up time = number of cavity round-trips to reach a detectable signal
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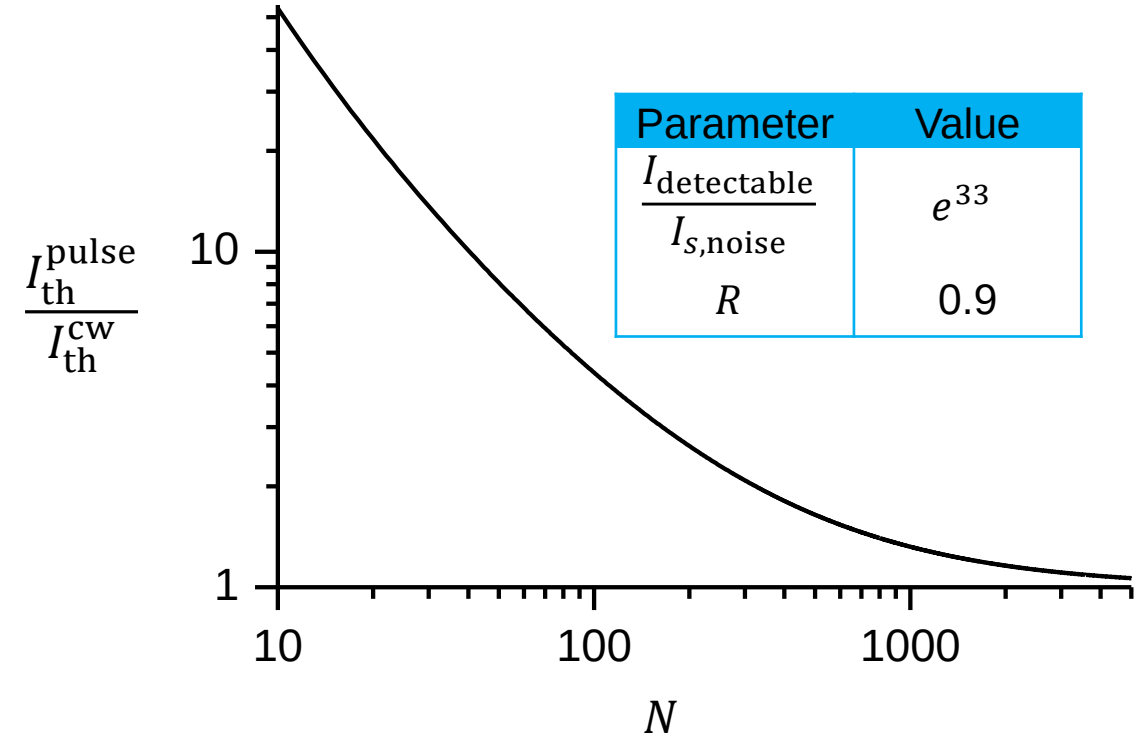
$$I_{s,N} = I_{s,noise} [R \cosh^2(\gamma L)]^N$$

$$t_{\text{build-up}}^{\text{SRO}} = N \times t_{\text{round-trip}}$$

Brosnan & Byer, IEEE J. Quantum Elec. 15 (1979) 415

Pulsed threshold

- Build-up time = pump pulse duration



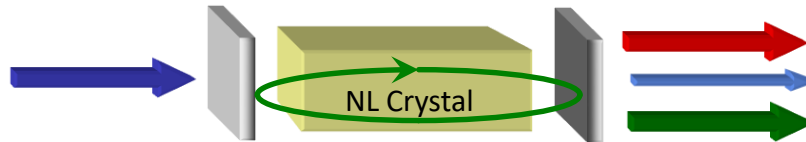
- The longer the pulse, the closer to the CW case
- Short cavity with less possible air spacing around the nonlinear crystal to minimize pulsed threshold

OPO in nanosecond regime

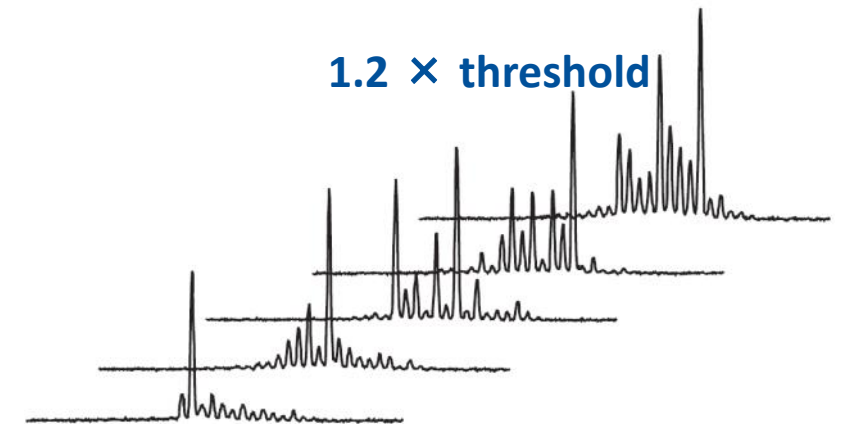
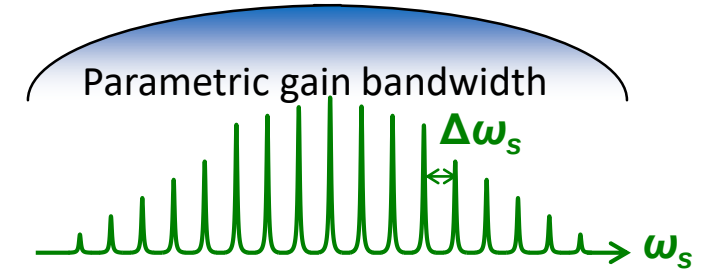
Issue of the control of the emitted spectrum

Simple singly “simply” resonant OPO

- Transient regime: mode competition does not fully develop → multimode emission
- Pulse-to-pulse variation of the energy distribution between the modes, related to the initial quantum noise → “noisy” multimode spectrum



➔ **Not relevant for gas detection applications**

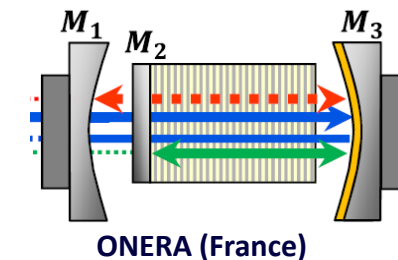
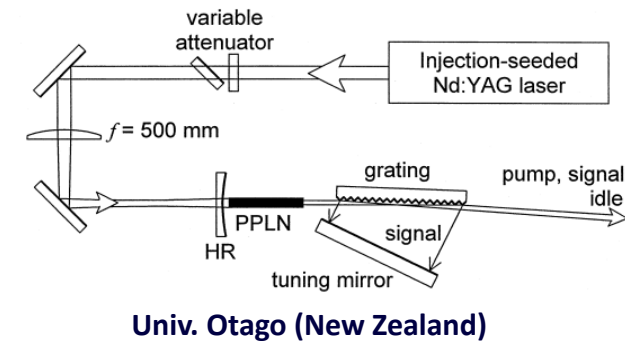
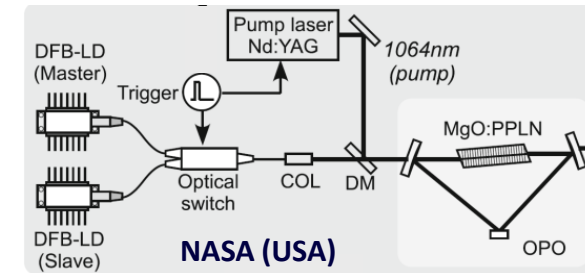


Pulse-to pulse spectrum
10-ns BBO OPO

Anstett and Wallenstein, Appl. Phys. B 79 (2004) 827-836

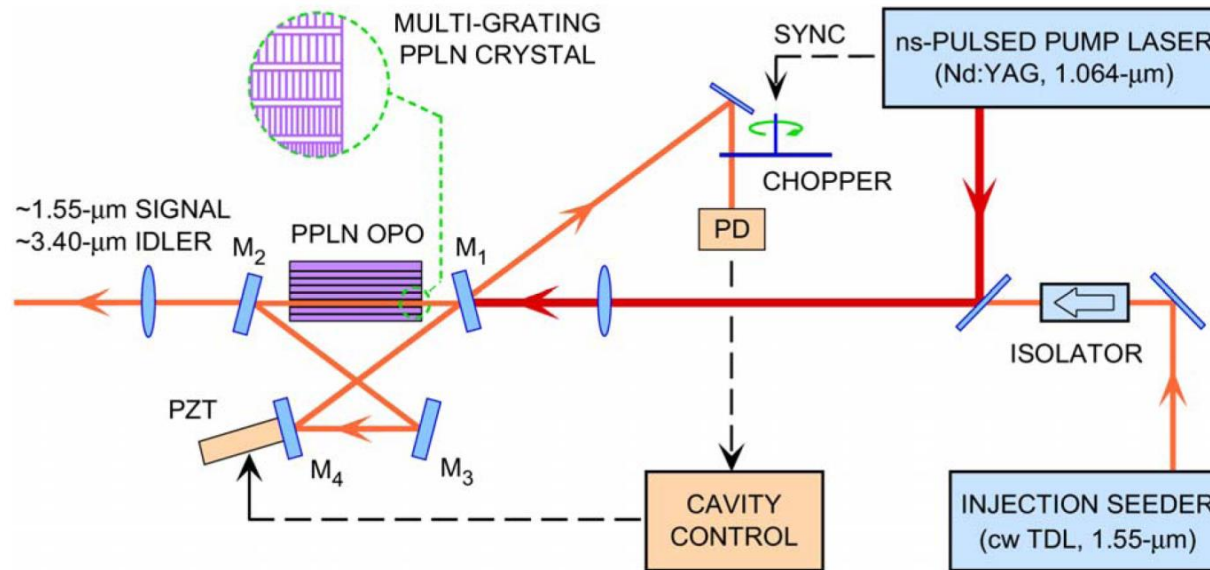
Main approaches for nanosecond OPOs line-narrowing

- Injection seeding by a single-frequency cw laser
- Insertion of intracavity wavelength selective elements
- Vernier spectral filtering with a dual cavity doubly resonant scheme



Injection seeding: principle

- A single-mode laser whose frequency is coincident with an OPO cavity mode is injected to seed the parametric oscillation*
 - Seed power $\gg$ quantum noise $\rightarrow$ oscillation builds up from this narrow-linewidth radiation
 - OPO cavity must be locked to the seed frequency for efficient injection



* Bjorkholm and Danielmeyer, Appl. Phys. Lett. 15 (1969) 171-173
Bell Labs, USA

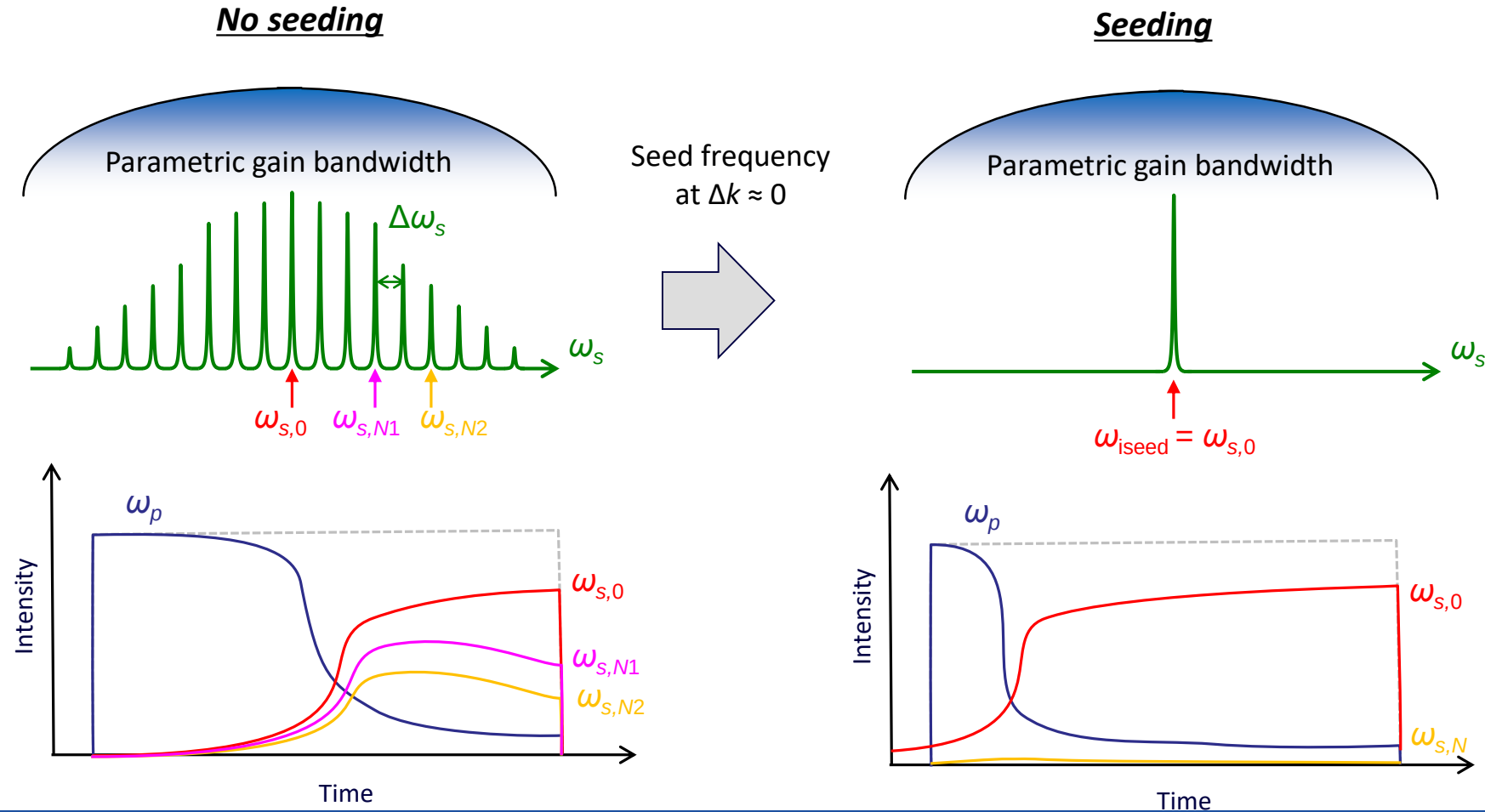


J. E. Bjorkholm
Source: Optica

Orr *et al*, Chapter 2 in *Tunable Laser Applications*, F. J. Duarte (ed) (2008) 15-96
Macquarie University (Australia)

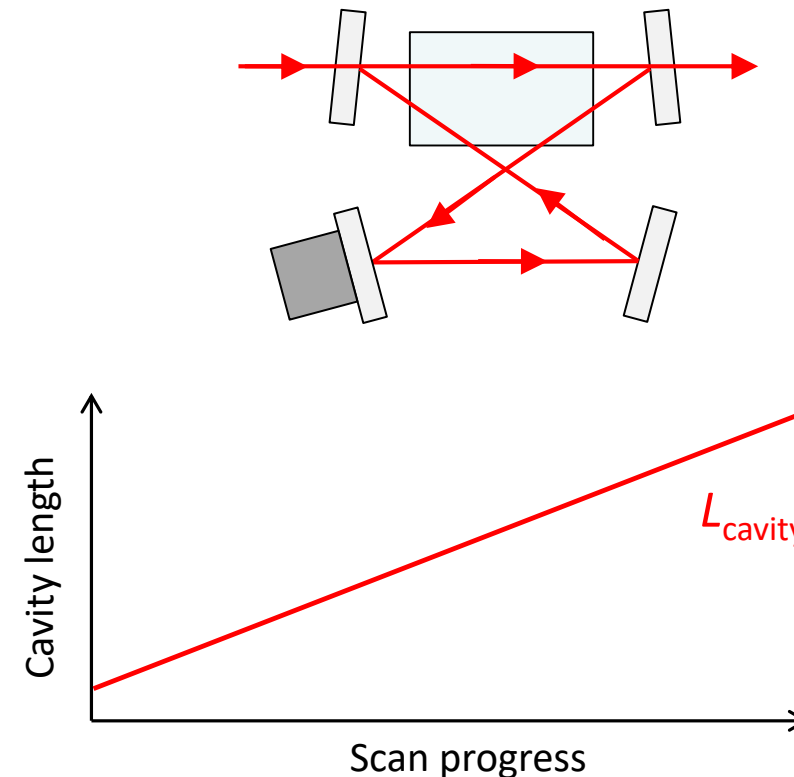
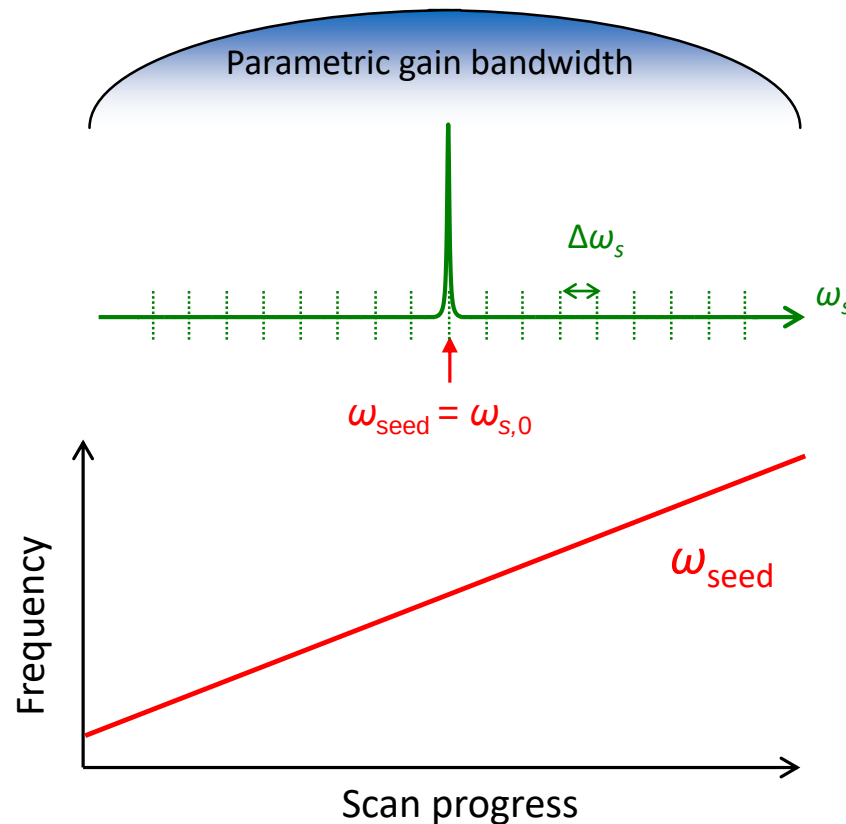
Injection seeding: effect of phase mismatching on spectral purity

- At $\Delta k \approx 0$, successful seeding $\rightarrow$ single-mode emission



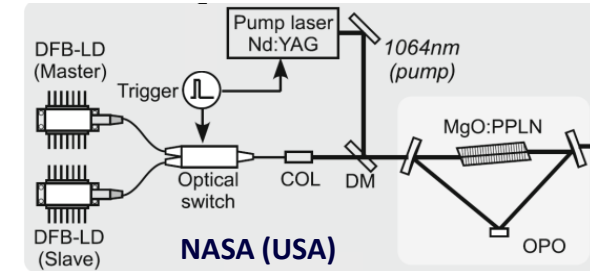
Injection seeding: fine tuning

- Simultaneous variation of the cavity length and tuning of the seed laser to maintain the coincidence between the cavity mode frequency and the seed frequency
- For broad tuning, phase matching has also to be adjusted to keep a successful seeding
- OPO tuning range is limited to seed laser tuning range

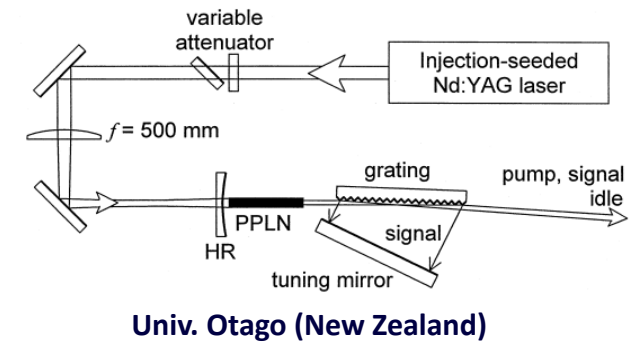


Main approaches for nanosecond OPOs line-narrowing

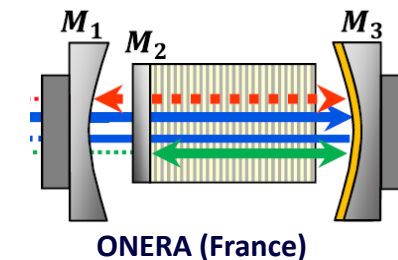
- Injection seeding by a single-frequency cw laser



- Insertion of intracavity wavelength selective elements



- Vernier spectral filtering with a dual cavity doubly resonant scheme



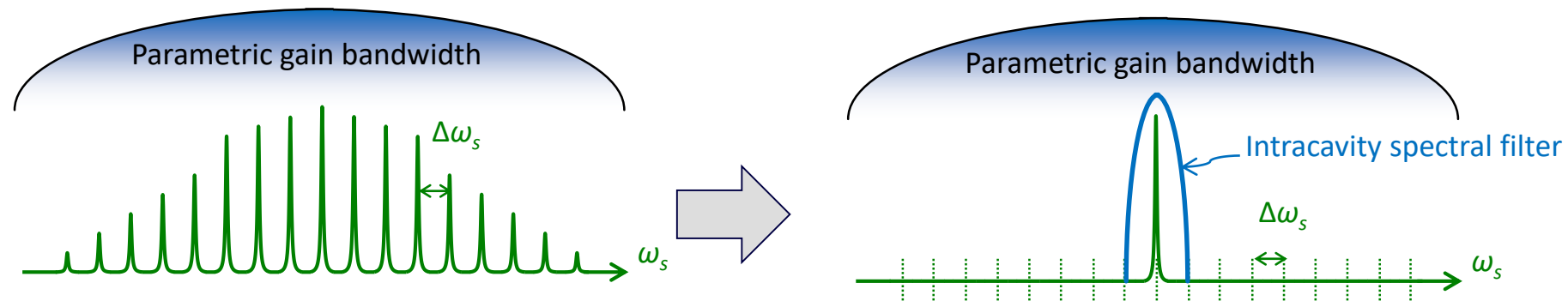
Line-narrowing with intracavity spectral filter: principle

- Insertion of a wavelength-dependent loss in the OPO cavity to select a single mode
 - Transposition of techniques previously implemented in dye lasers
 - First implementations in the late 60's-early 70's

Kreuzer, Appl. Phys. Lett. 15 (1969) 263-265
Bell Labs (USA)



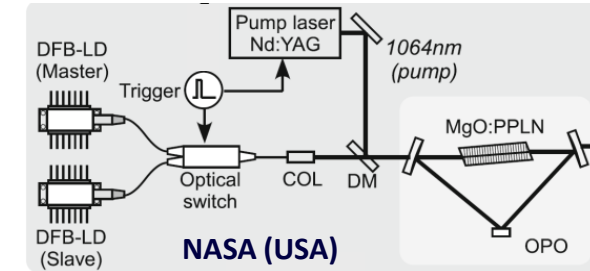
L. B. Kreuzer
Source: The Rayleigh Register



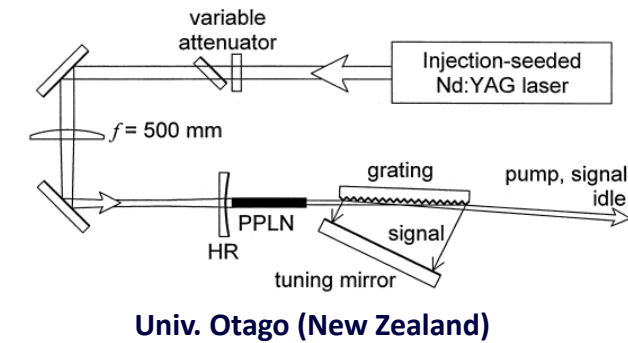
- The spectral filter induces loss for unwanted modes
- Increase of the oscillation threshold (typically $\times 2$)
- Reduced efficiency
- Complexity of implementation requiring precise control of several elements of the cavities

Main approaches for nanosecond OPOs line-narrowing

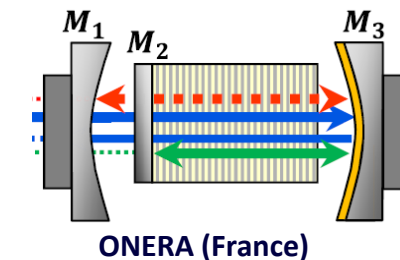
- Injection seeding by a single-frequency cw laser



- Insertion of intracavity wavelength selective elements

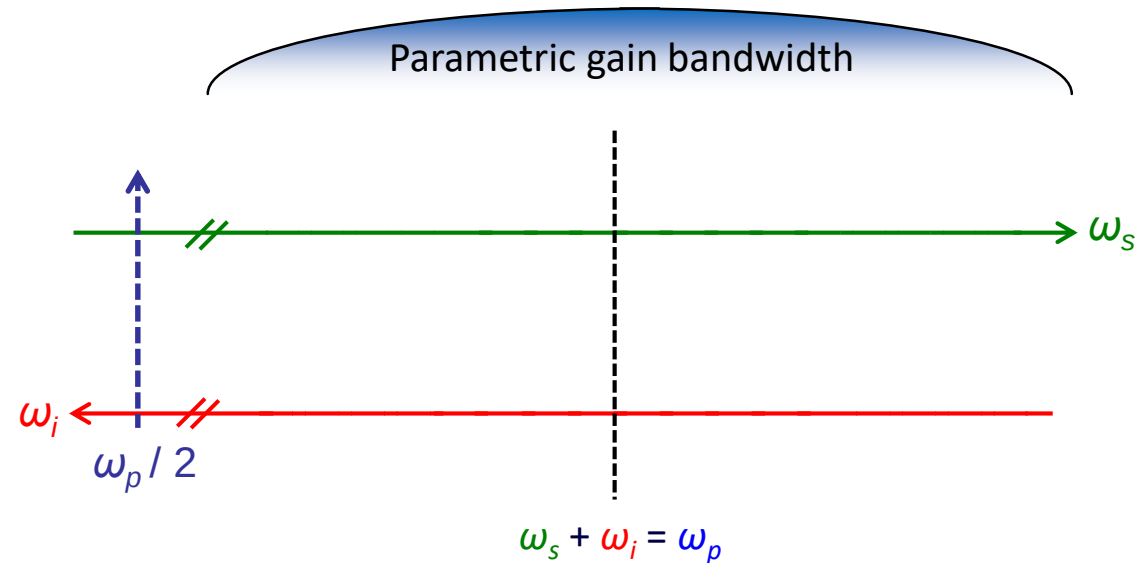
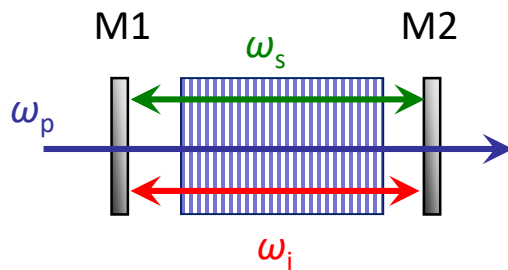


- Vernier spectral filtering with a dual cavity doubly resonant scheme



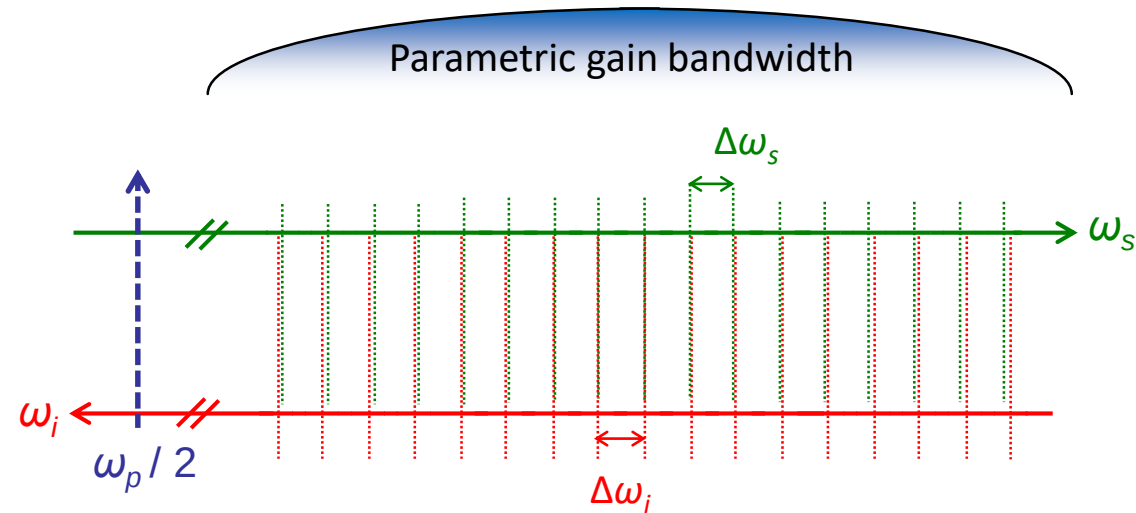
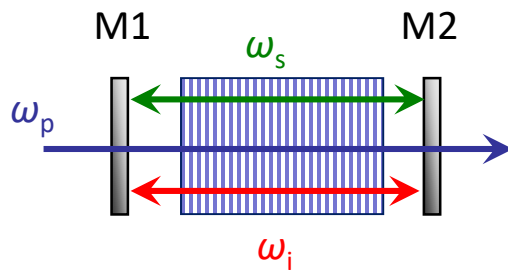
Spectral content of a pulsed doubly resonant OPO

- Single-cavity doubly resonant OPO (DRO):
 - Giordmaine and Miller's diagram: signal and idler frequencies located on the same vertical satisfy $\omega_s + \omega_i = \omega_p$



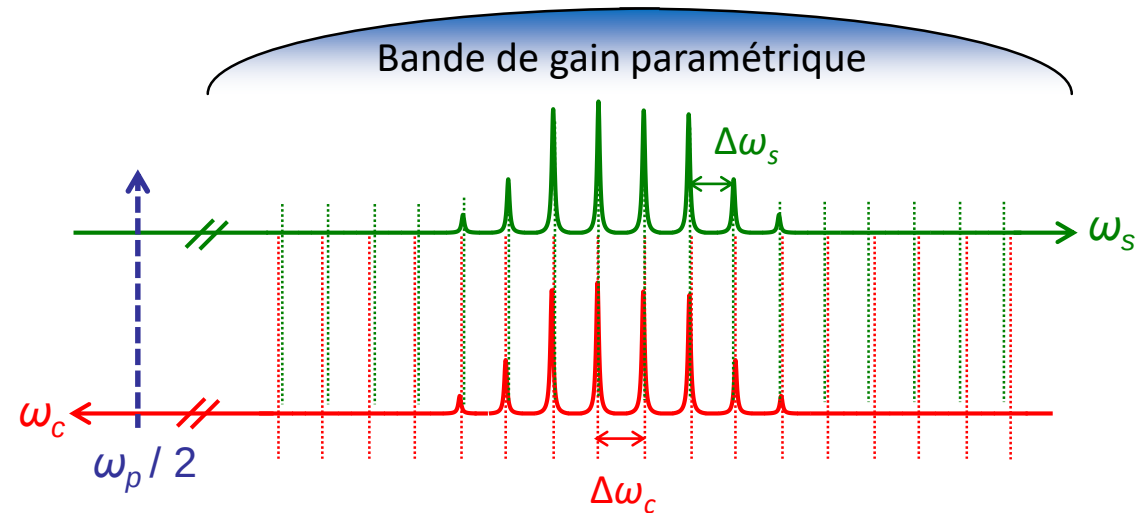
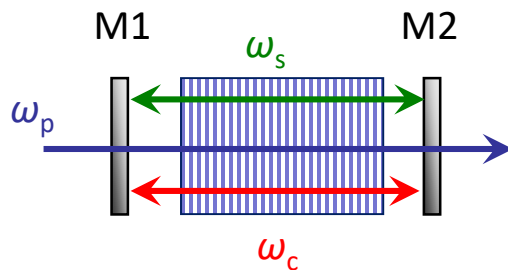
Spectral content of a pulsed doubly resonant OPO

- Single-cavity doubly resonant OPO (DRO): $\Delta\omega_s \approx \Delta\omega_c$
 - Giordmaine and Miller's diagram: signal and idler frequencies located on the same vertical satisfy $\omega_s + \omega_i = \omega_p$
 - Chromatic dispersion and/or birefringence in the nonlinear crystal $\rightarrow$ FSRs are slightly different for signal and idler



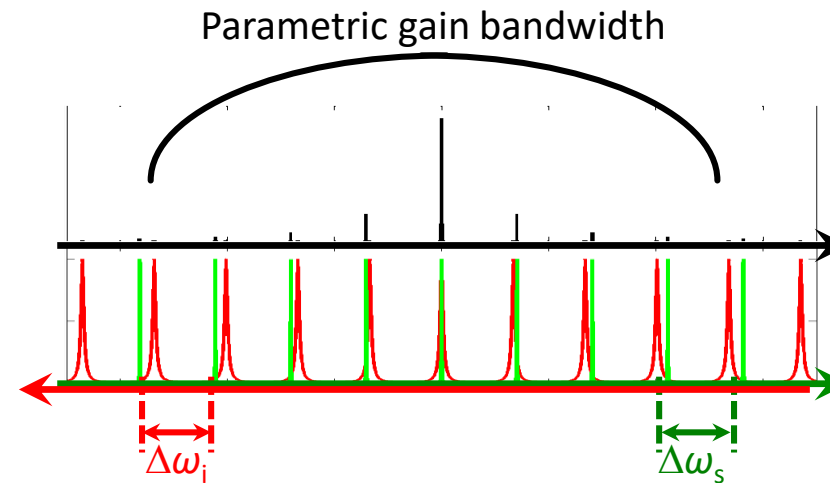
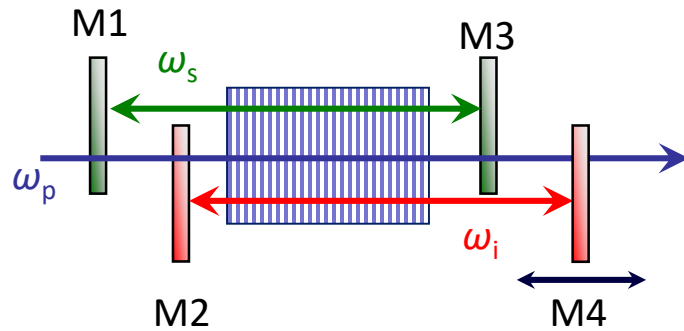
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 - Chromatic dispersion and/or birefringence in the nonlinear crystal $\rightarrow$ FSRs are slightly different for signal and idler
 - Partial coincidences of signal-idler mode pairs however occur close to a signal-idler mode pair in exact coincidence $\rightarrow$ a pulsed single-cavity DRO is usually multimode because these partial coincidences sufficiently overlap to be able to oscillate, forming a so-called cluster of modes



Dual-cavity DRO in nanosecond regime

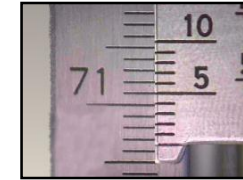
- DRO with separate signal and idler cavities:
 - Cavity lengths can be adjusted independently



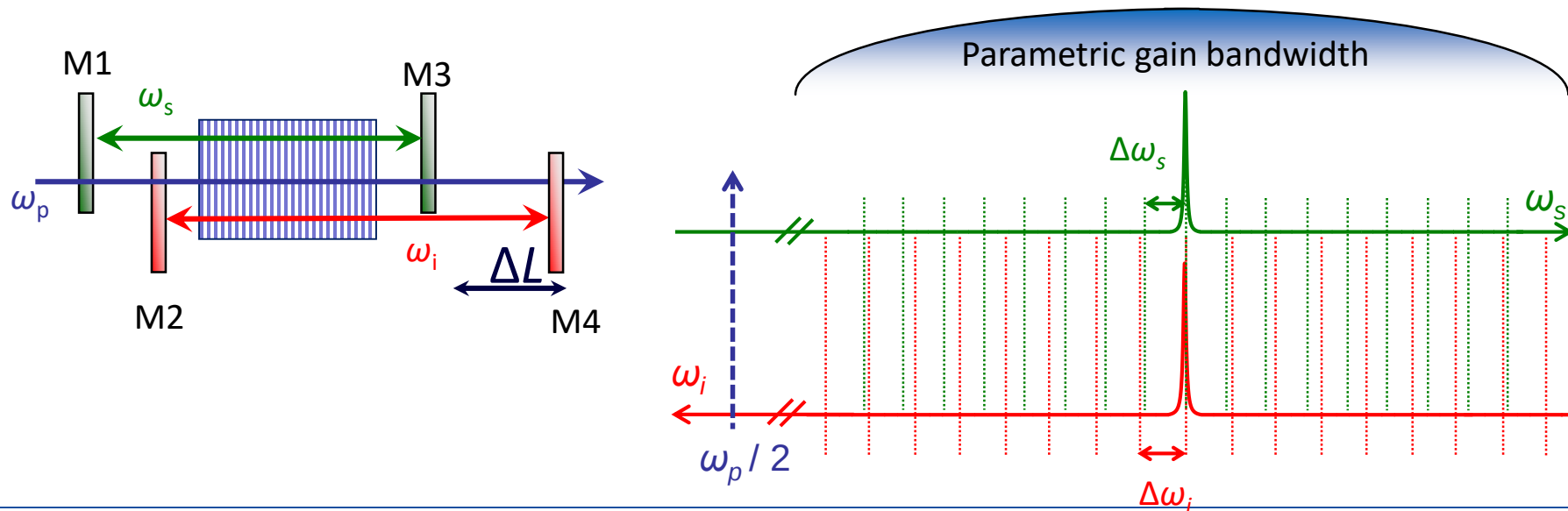
Vernier spectral filtering in pulsed dual-cavity DROs

- Dual-cavity doubly resonant OPO (DRO): $\Delta\omega_s \approx (1 - \Delta L/L) \Delta\omega_i$
 - Cavity lengths can be adjusted independently
 - Proper adjustment of the cavity dissociation $\rightarrow$ a single signal-idler mode pair can oscillate $\rightarrow$ single-mode emission
 - Other pairs of modes do not sufficiently overlap to oscillate
 - No insertion loss $\rightarrow$ low oscillation threshold

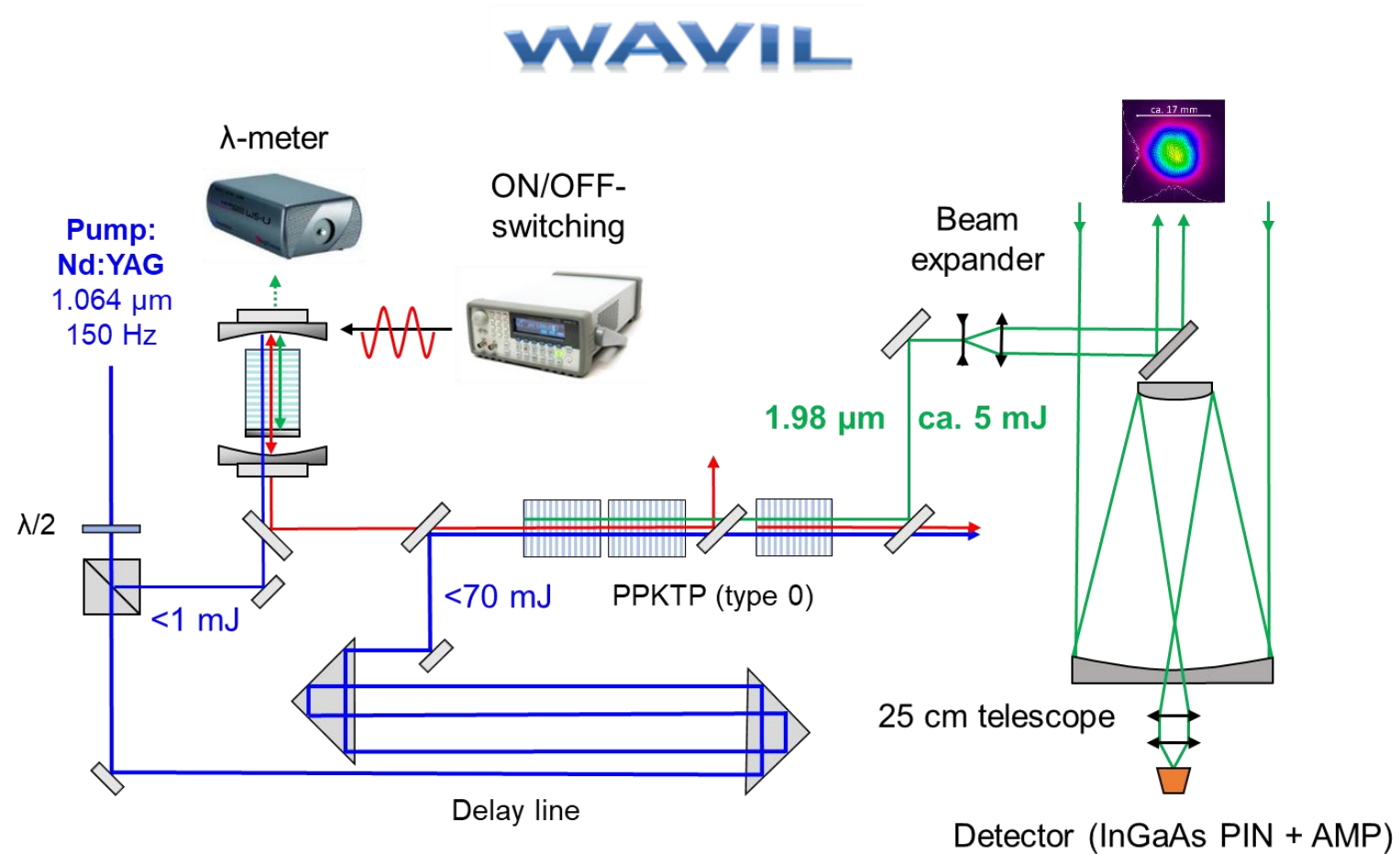
Scherrer *et al*, J. Opt. Soc. Am. B 17 (2000) 1716
ONERA (France)



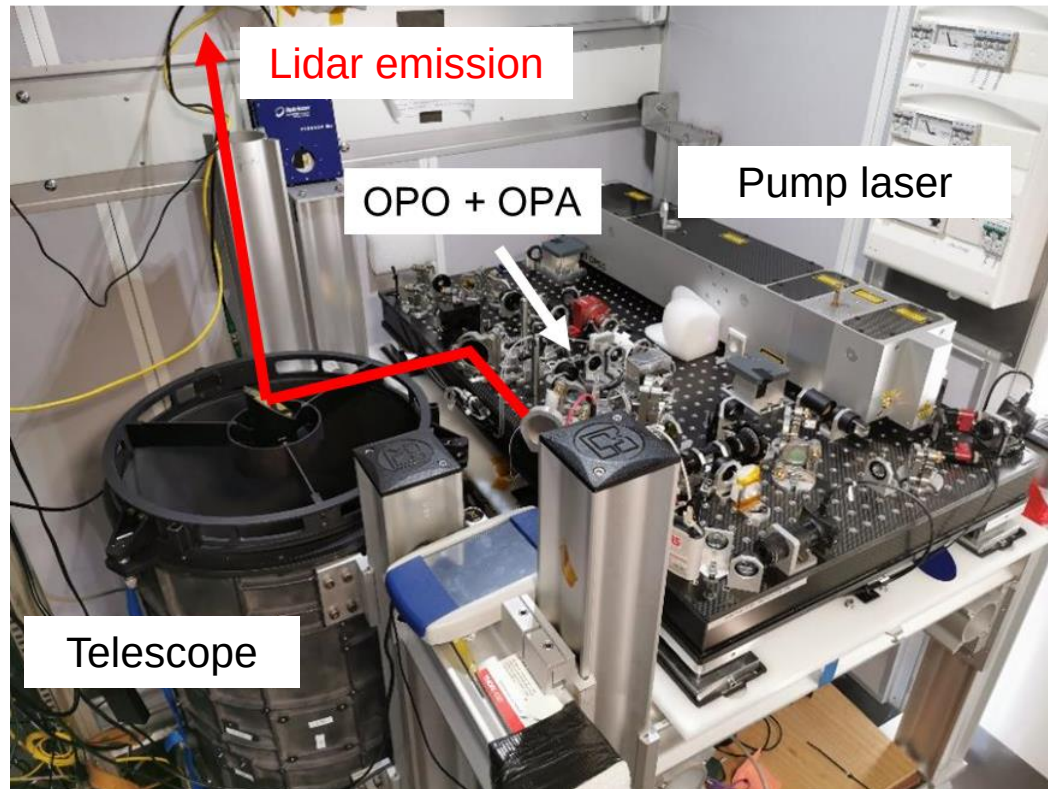
M. Lefebvre
Source: ONERA



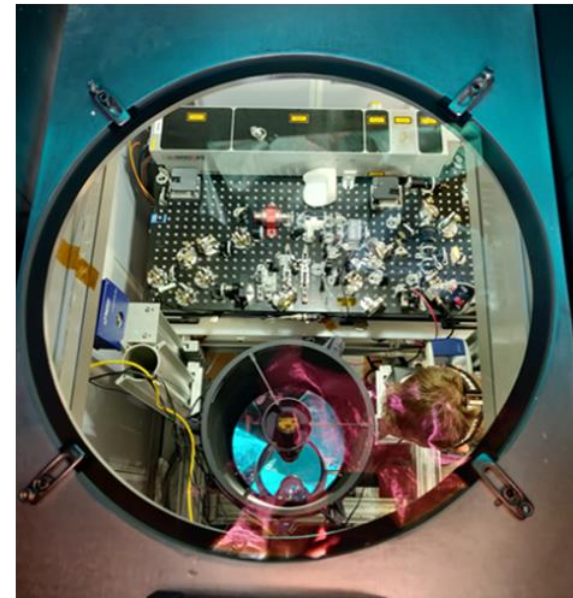
Application example: water vapour and isotopes (H₂O/HDO) LIDAR at 1982 nm



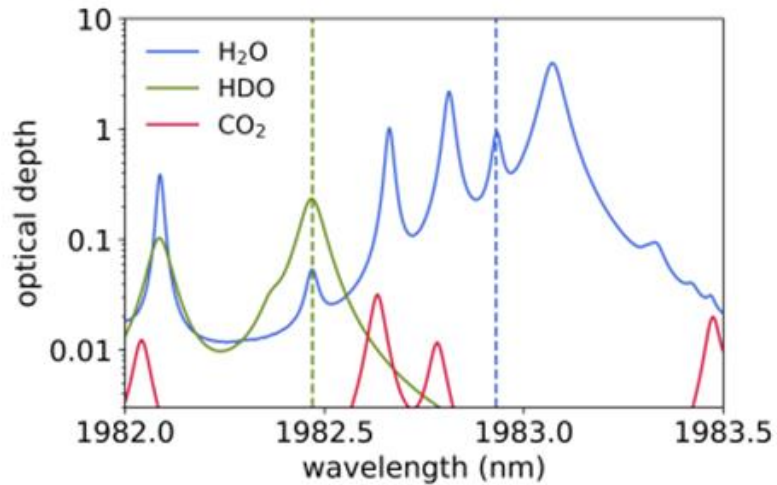
Application example: water vapour and isotopes (H₂O/HDO) LIDAR at 1982 nm



WaVIL lidar integrated into LSCE's instrumented truck

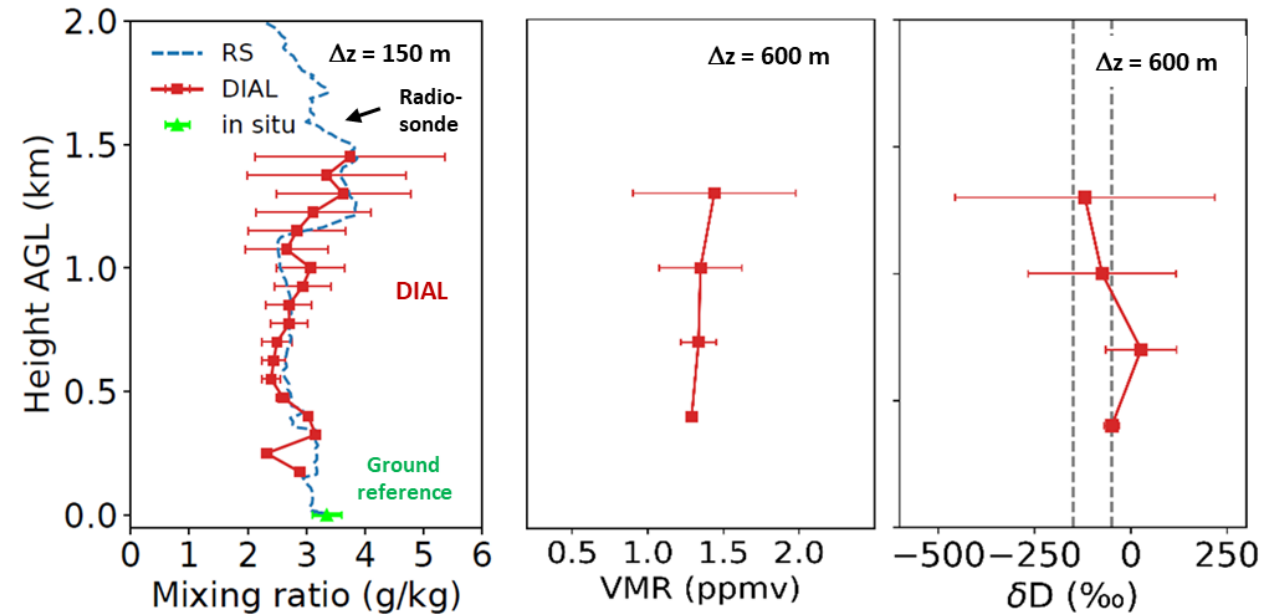


Application example: water vapour and isotopes (H₂O/HDO) LIDAR at 1982 nm



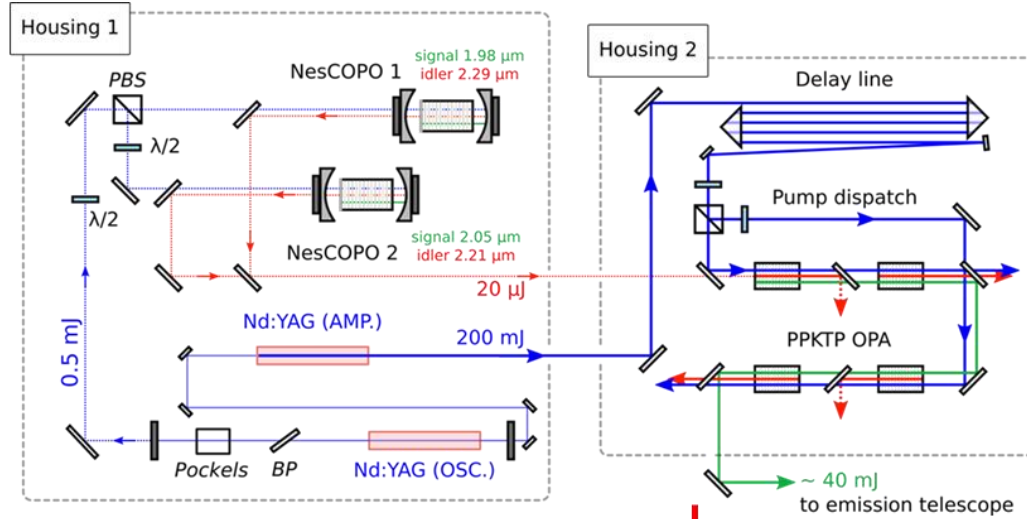
* J. Hamperl, et al., Opt. Express, OE 30, 47199–47215 (2022).

WAVIL

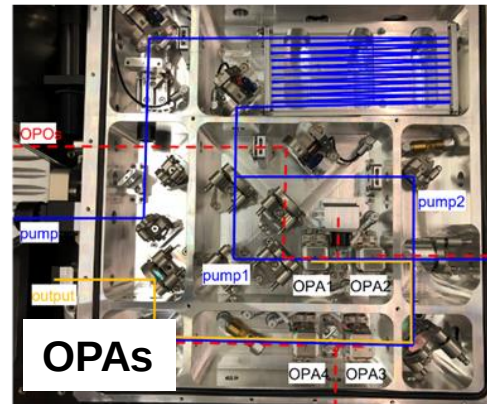
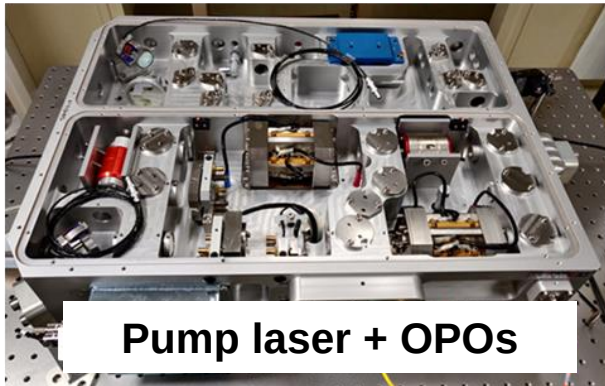
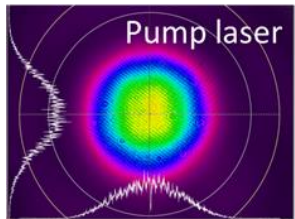
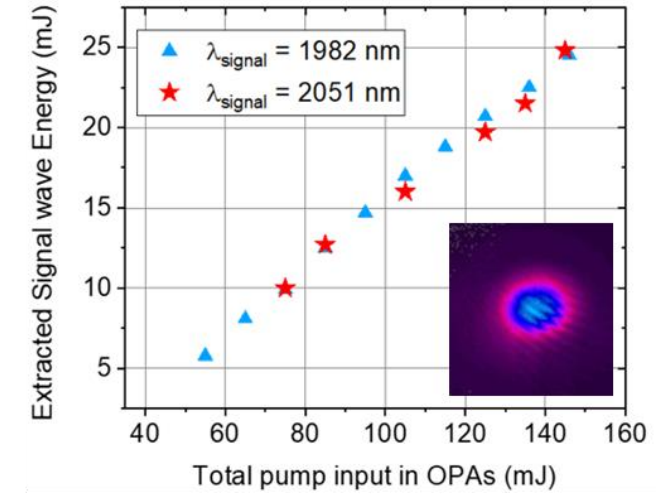


- ❑ 25-min integration time (150 Hz)
- ❑ Issue of HDO measurement sensitivity

Technological maturation and on-board systems

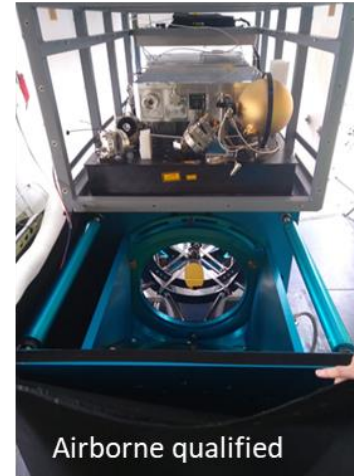
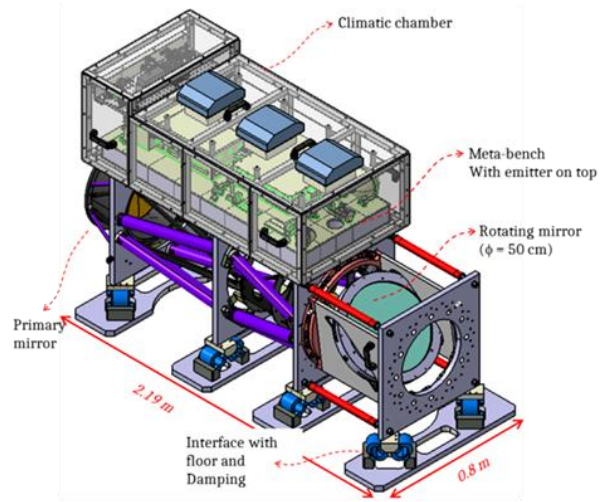


5 × 5 mm²
aperture
PPKTP crystals



- ❑ Design and integration ~ 3 years
- ❑ Up to 30% efficiency
- ❑ $M^2 < 1.6$
- ❑ 25 mJ for signal wave alone

Technological maturation and on-board systems



- ❑ Telescope: 30 cm
- ❑ Detector: InGaAs PIN + TIA (BW: 3.5 MHz, NEP = 1.3 pW/VHz)
- ❑ Emission: 10 ns, 10 mJ, 75 Hz (2 × pulse)

Main approach for nanosecond OPOs line-narrowing

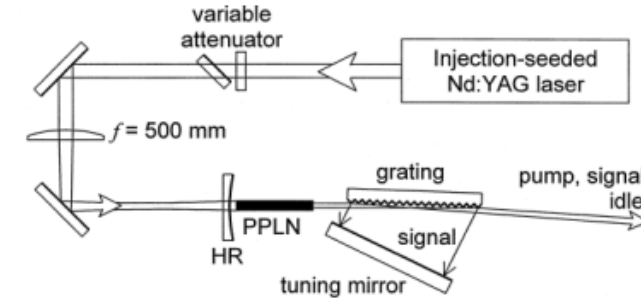
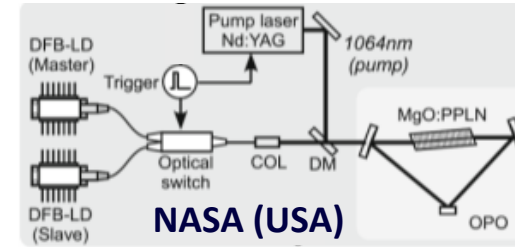
- Injection seeding by a single-frequency cw laser
- Insertion of intracavity wavelength selective elements
- Vernier spectral filtering with a dual cavity doubly resonant scheme

➡ ...still have to deal with high mechanical stability requirements

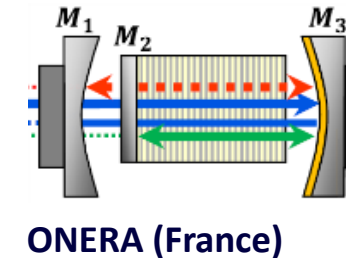


A well-known solution from the laser world is the **DFB laser**

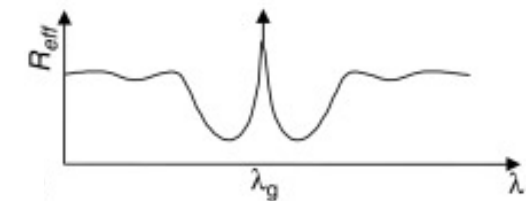
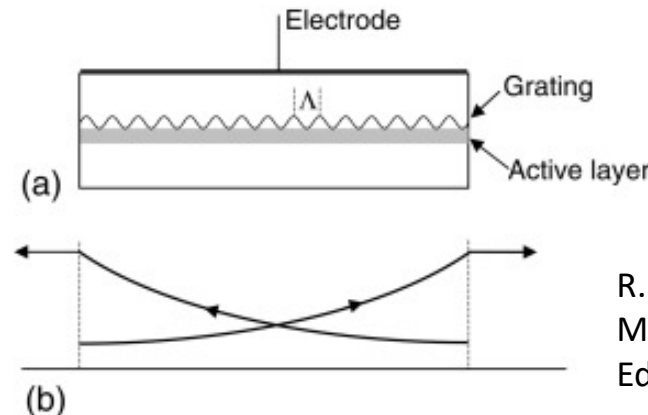
- No cavity
- Single frequency emission
- Fine wavelength tuning



Univ. Otago (New Zealand)



ONERA (France)

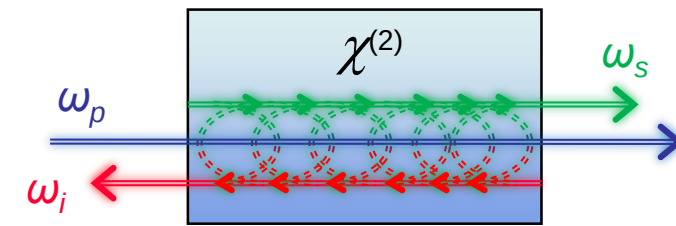


R. Hui, M. O'Sullivan, Fiber-Optic Measurement Techniques (Second Edition), Academic Press, 2023

Backward-wave OPO (BWPO)

Nonlinear analog of the distributed feedback (DFB) laser

- QPM condition such as one of the generated wave is contra-propagating
- Two counter-propagating parametric waves are present → positive feedback → oscillation
- No need of any additional cavity → mirrorless device
- Intrinsically narrow-band
- Proposed by Harris in 1966



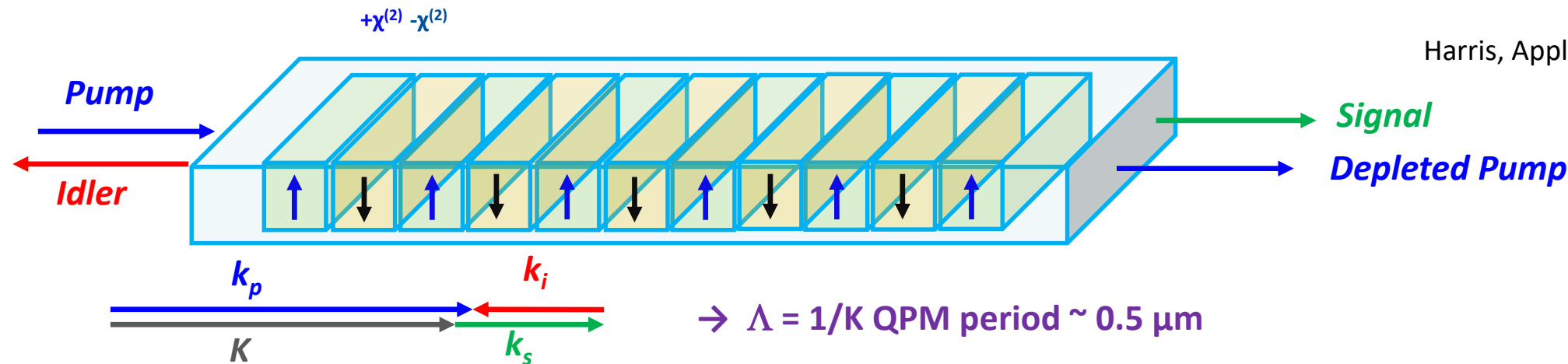
PROPOSED BACKWARD WAVE OSCILLATION IN THE INFRARED*

(parametric oscillation; birefringence; internal feedback;
nonlinear optics; T)

S. E. Harris
Department of Electrical Engineering
Stanford University, Stanford, California
(Received 13 June 1966)



Harris, Appl. Phys. Lett. 9 (1966) 114–116



- Short-period periodically poled Rb:KTiOPO₄ → first experiment reported by Canalias and Pasiskevicius in 2007 at KTH

Canalias & Pasiskevicius, Nature Photon 1 (2007) 459–462

Backward-wave OPO pump tuning

Main λ tuning lever = pump tuning

$$\frac{\partial \nu_f}{\partial \nu_p} = 1 + \varepsilon$$

and

$$\frac{\partial \nu_b}{\partial \nu_p} = -\varepsilon$$

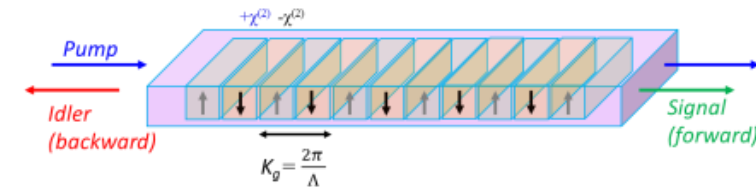
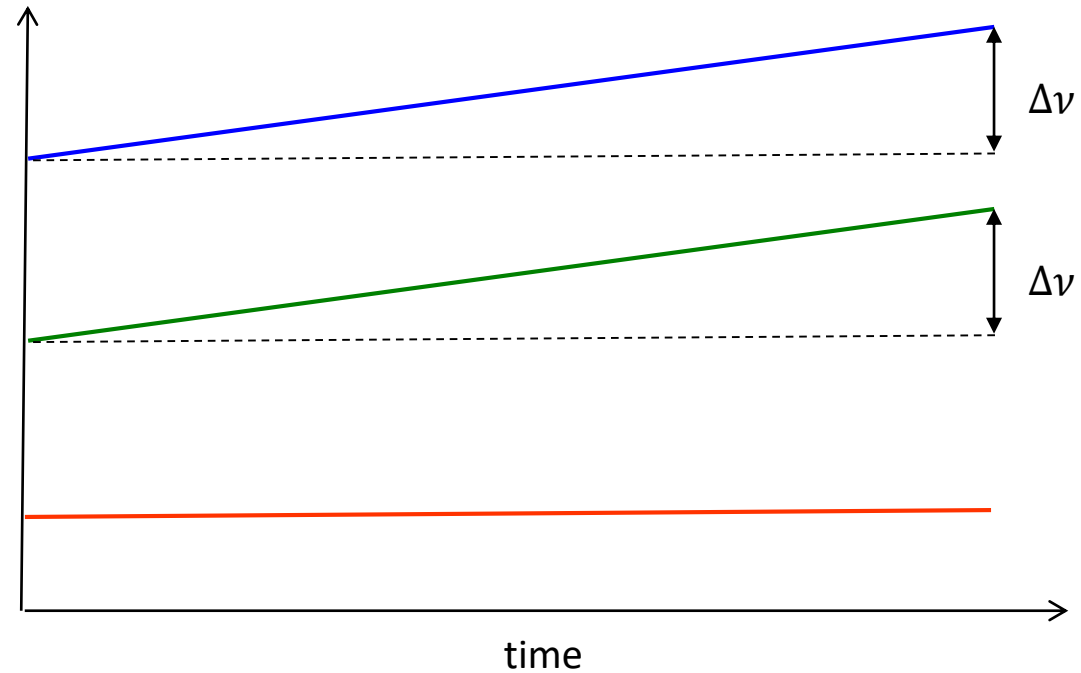
$$\varepsilon \sim 10^{-3}$$



ν_p

ν_f

ν_b



Canalias & Pasiskevicius, Nature Photon 1 (2007) 459–462



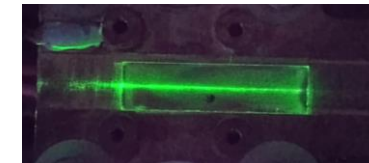
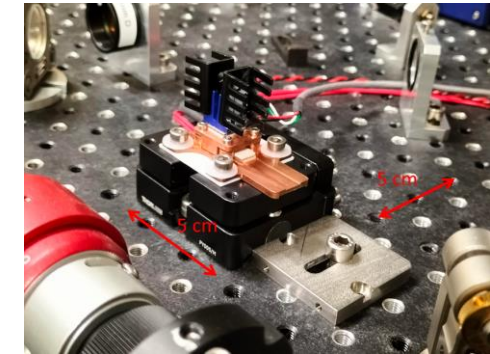
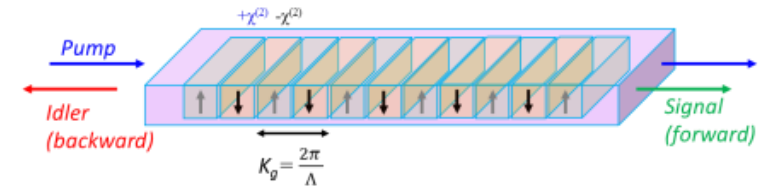
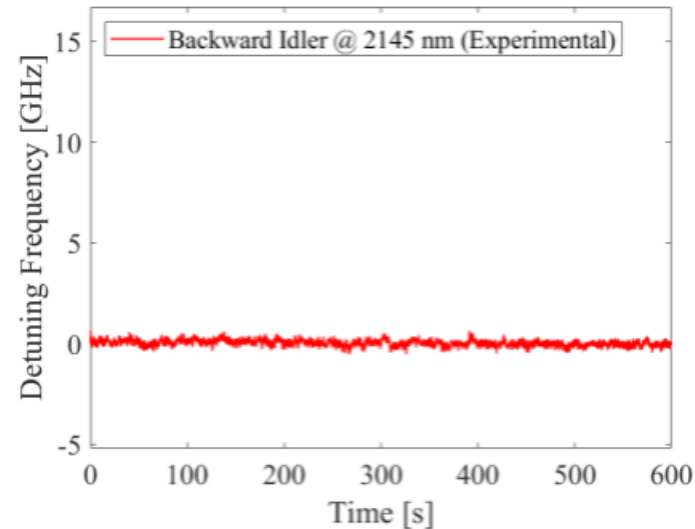
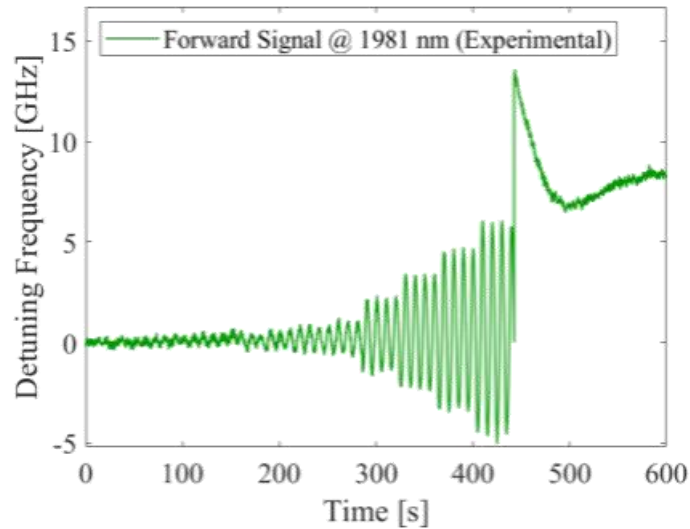
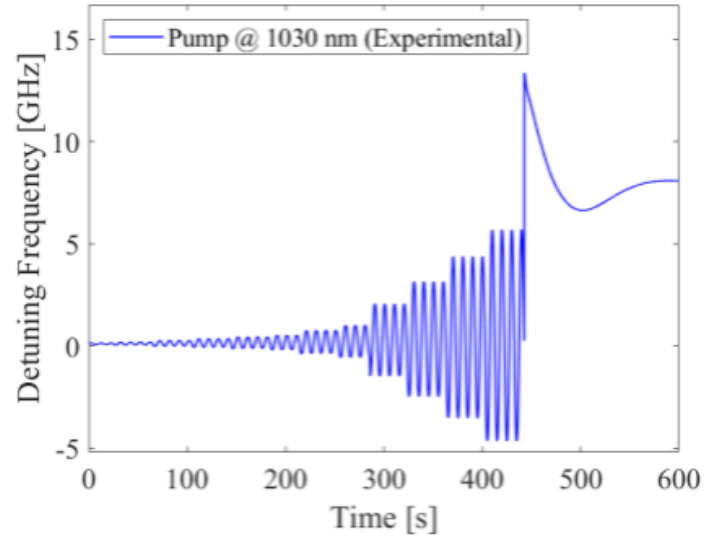
Forward : Spectrally shifted pump replica

- Fourier transform (FT) limited if pump is FT limited



Backward : FT limited

Pump-tuning: experiment



23 mm long
BWPO

Mølster *et al.*, Opt. Lett. (2023) 6484

Nonlinear parametric optics

1. Principles of three-wave mixing

- Nonlinear response
- Difference-frequency generation: conversion efficiency and acceptance
- Phase- and quasi-phase-matching

2. Parametric amplification, generation, and oscillation

- Parametric amplification and reconversion effects
- Parametric fluorescence
- Parametric oscillation, variations of OPO schemes

3. Implementations and related applications

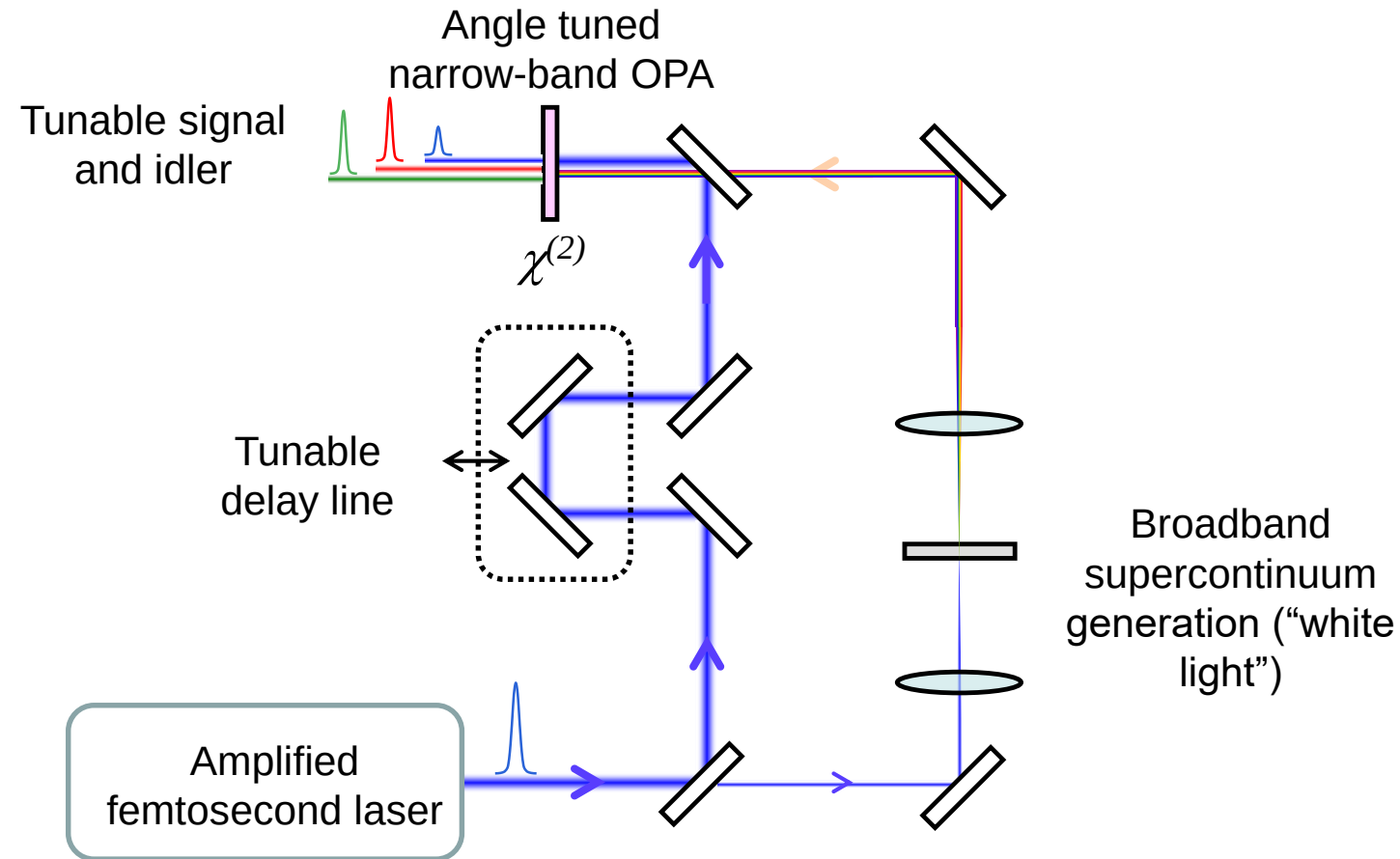
- Continuous-wave regime
- Nanosecond regime, spectral content issues
- Picosecond to femtosecond regime

Conclusion

Parametric sources in pico-femtosecond regime

Single-pass yield is sufficient to implement efficient sources

Typical scheme

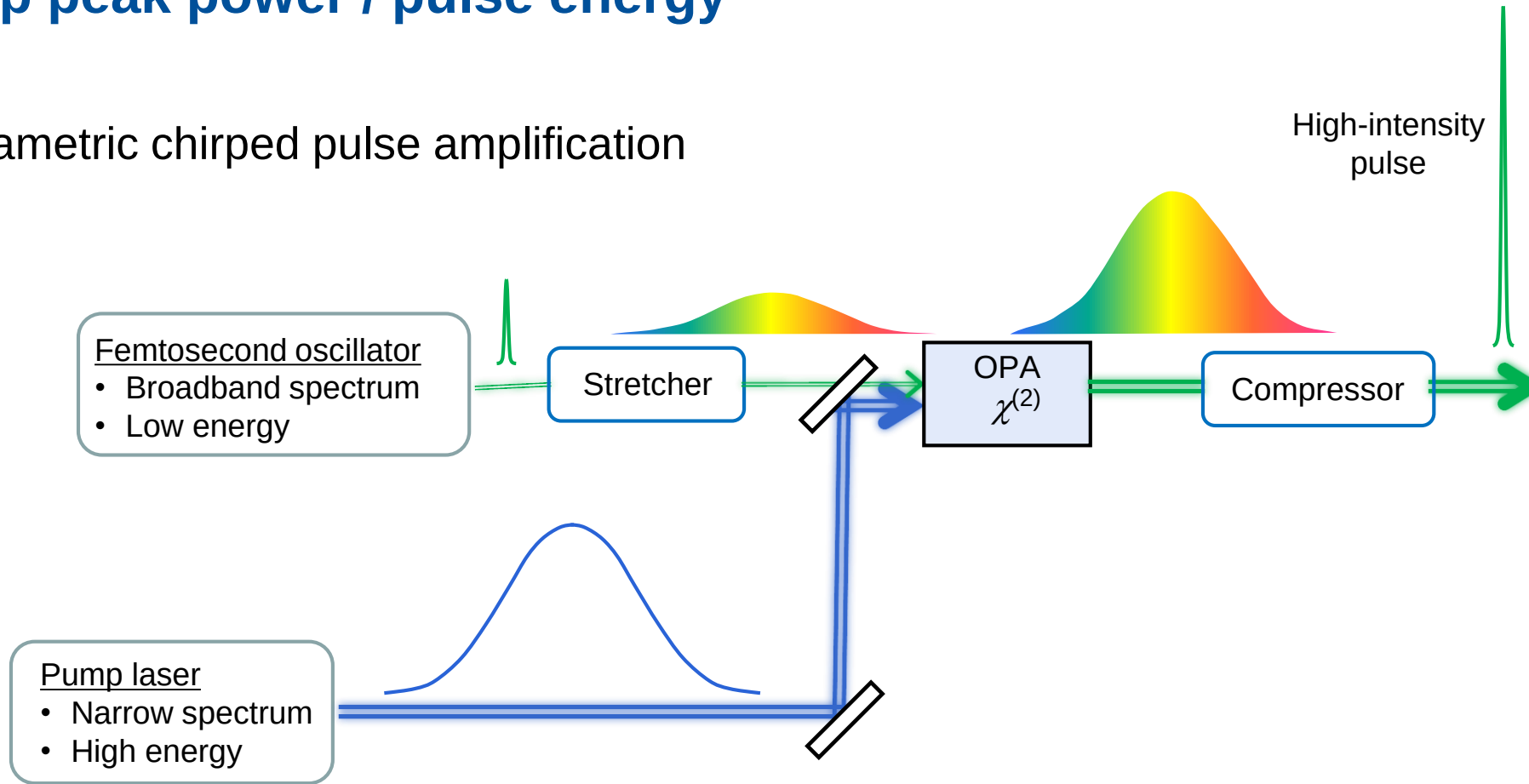


- Used for various applications including pump-probe spectroscopy, fundamental physics, nonlinear microscopy, optical diagnostics (reactive-flow thermometry)

Scaling up peak power / pulse energy

OPCPA

Optical parametric chirped pulse amplification



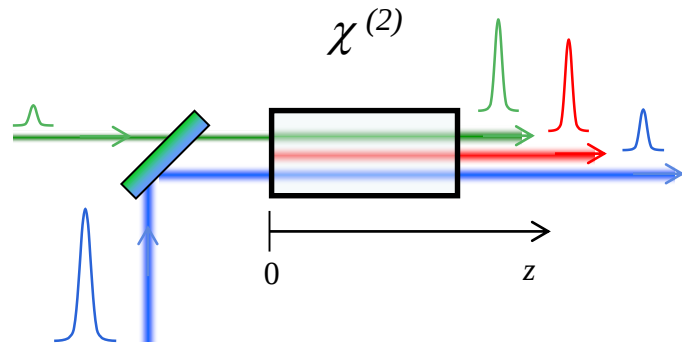
- Enables to reach very high peak power (terawatt, petawatt)
- Used in high-field physics, high-order harmonic generation (HHG) → attophysics*

*Yesterday's presentation by Anne L'Huillier

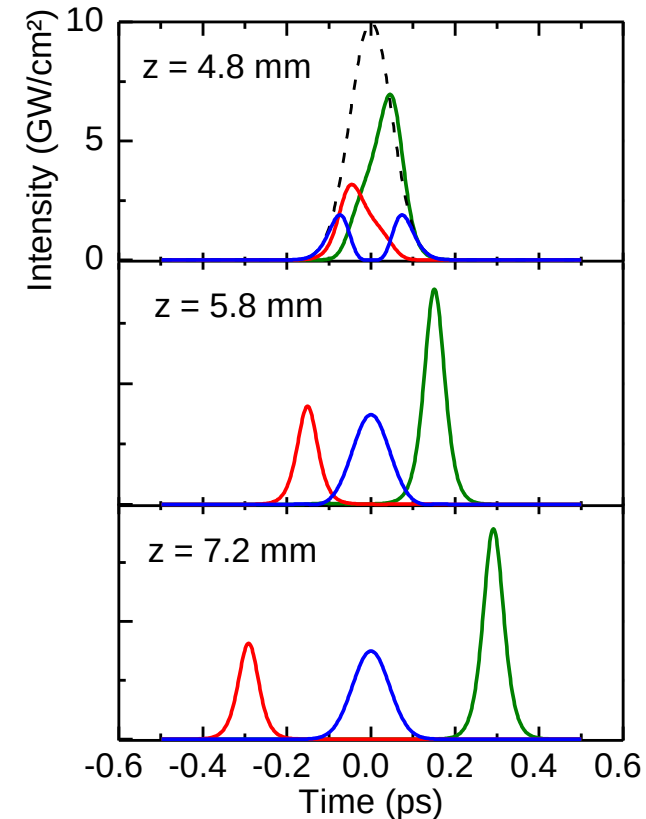
Group velocity effects

Pulses are shorter than crystals

- $1 \text{ ps} \times c = 300 \text{ }\mu\text{m}$; $10 \text{ fs} \times c = 3 \text{ }\mu\text{m}$
- The pulses shift relative to each other as they propagate through the nonlinear crystal due to differences in group velocity, this is referred as **temporal walk-off**



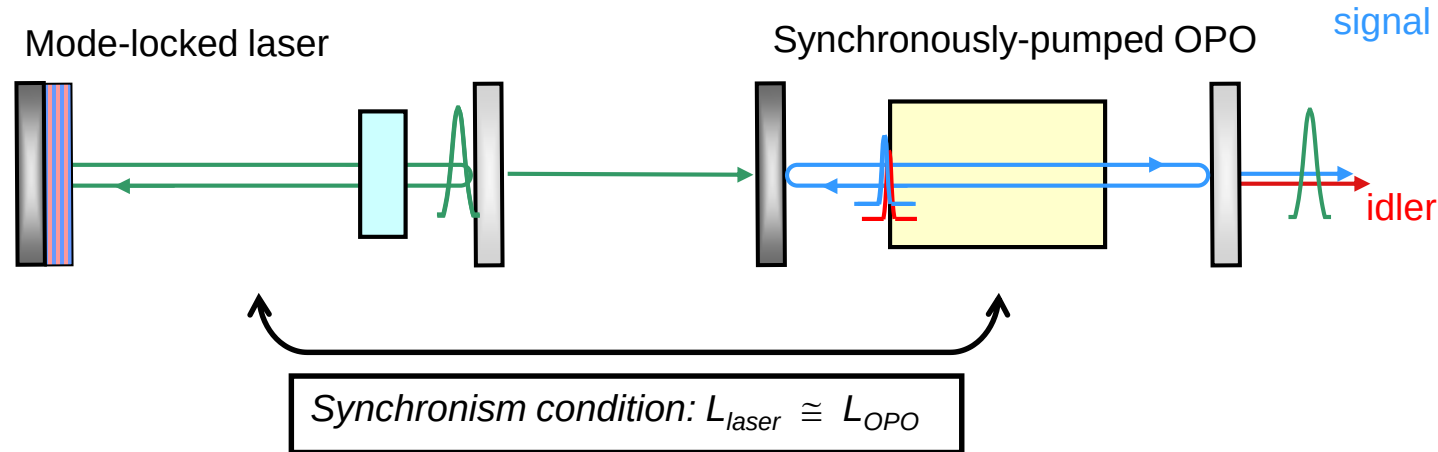
Parameter	Value	Parameter	Value
λ_p	$1.03 \text{ }\mu\text{m}$	χ_{eff}	20 pm/V
λ_s	$1.5 \text{ }\mu\text{m}$	n	1.6
λ_i	$3.29 \text{ }\mu\text{m}$	$n_{g,p}$	1.63
I_p	10 GW/cm^2	$n_{g,s}$	1.66
I_s	10 mW/cm^2	$n_{g,c}$	1.6



➡ What if we want to make a short-pulse OPO?

OPO in picosecond-femtosecond regime

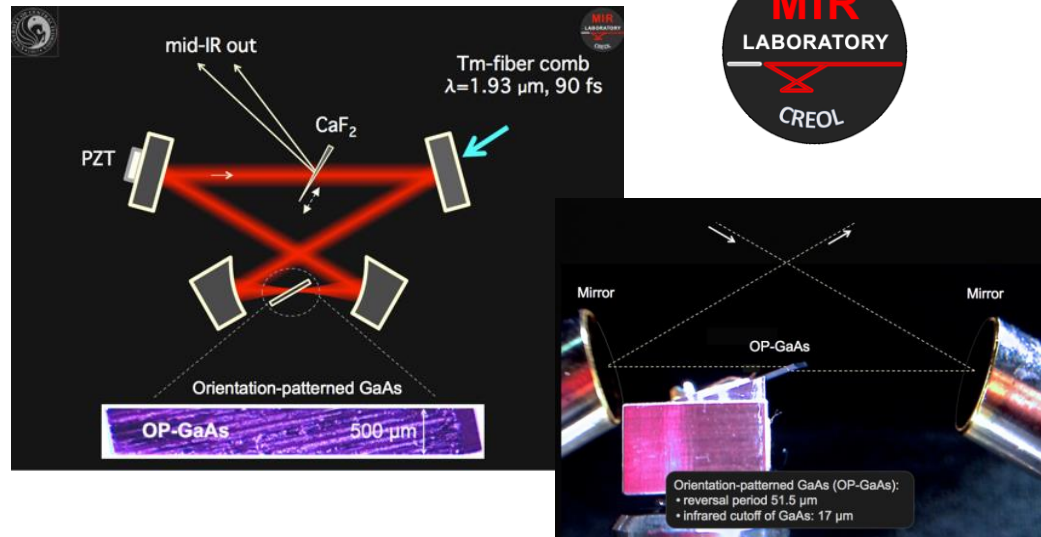
Principle of synchronous pumping:



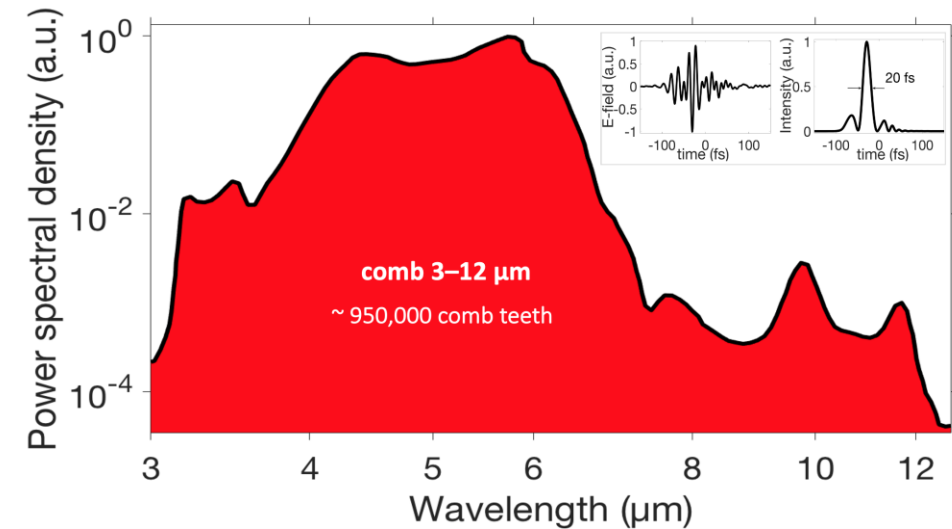
- Picosecond regime: a simple mechanical adjustment of the OPO cavity length is usually sufficient
- Femtosecond regime: A (piezo) servo-locking is generally required to ensure proper operation
- Fairly good technological maturity, several commercial products available

Synchronously-pumped OPO at degeneracy

Generation of an ultra-broadband octave-spanning spectrum



Ru et al, Opt. Lett. 46 (2021) 709 : last result : 3-12 μm , i.e. 2 octaves



- Frequency comb
- Mutual-coherence properties with the pump field
- Applications in spectroscopy and metrology
- Can be duplicated for dual-comb spectroscopy
- Also used for random number generation and Ising machine

Nonlinear parametric optics

1. Principles of three-wave mixing

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- Difference-frequency generation: conversion efficiency and acceptance
- Phase- and quasi-phase-matching

2. Parametric amplification, generation, and oscillation

- Parametric amplification and reconversion effects
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3. Implementations and related applications

- Continuous-wave regime
- Nanosecond regime, spectral content issues
- Picosecond to femtosecond regime

Conclusion

Conclusion

- Parametric sources enable to generate coherent light in spectral ranges that are poorly or not covered by direct laser emission lasers or in a different temporal regime
- They can cover temporal regimes from continuous-wave emission to ultrashort pulses
- Today, it is a unique device to control the properties of coherent light in any part of the spectrum, with a sufficient optical-optical efficiency
- These unique properties make them highly relevant for various applications such as gas sensing or infrared countermeasures as well as for fundamental science spanning from quantum optics to strong-field physics